

Noise in Radiographic Imaging

Review: Sources of Noise

- 1) Systematic
 - Geometric distortion
 - Non-linearity of detector
 - Sampling error
 - Computation (eg Reconstruction) error
- 2) Random
 - Photon noise
 - Film grain noise
 - Electronic Noise
 - Scattered Radiation

Noise Characterization

- Objective physical Characterization
 - Probability density
 - SNR
 - Power Spectrum
 - DQE
- Subjective Characterization
 - Measure of performance or
 - Psychophysical evaluations or
 - correctness of Diagnosis
 - TP
 - TN
 - FP
 - FN

Wiener Spectrum with a Film-Screen system

- Assuming Stationary process means:
x-ray exposure has an average flux of density which is constant over area of the screen.

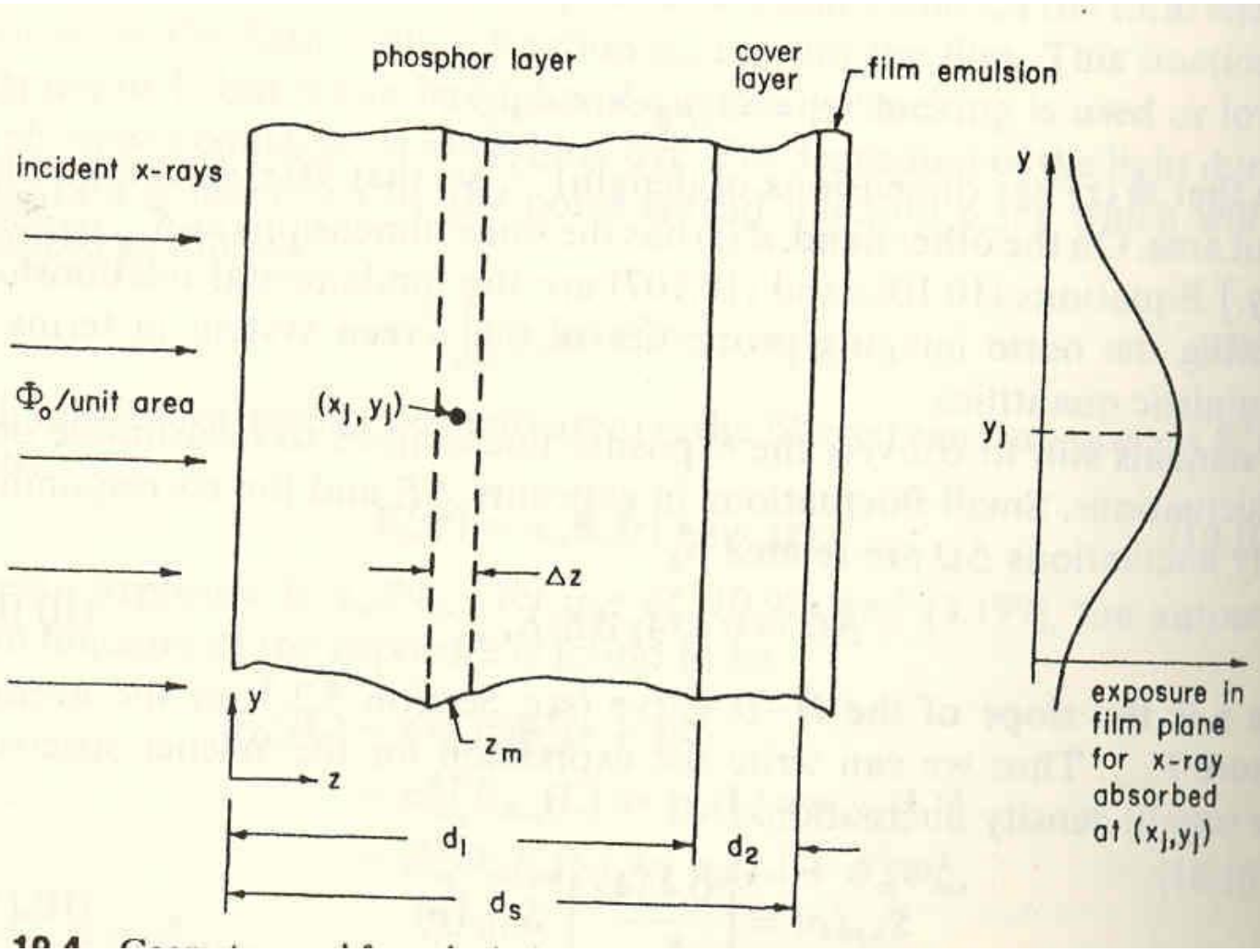
Therefore, Fluctuation in film exposure and film optical density is described by Power Spectral Densities (or Wiener Spectrum)

It is also called Noise Transfer Properties of the Film-Screen

Film-Screen Response

- The input is an x-ray flux and the output is a fluctuating density distribution.
- Overall system Response is thus:
 - 1) Converting from x-ray to an input irradiance distribution
 - 2) performing a filtering process which is spreading of a flush of light from a point source (where x-ray is absorbed in phosphor)
 - 3) Converting the filtered exposure fluctuation to optical density fluctuations on film (where light is absorbed to the emulsion grains)

Film-Screen Interaction Model



input irradiance distribution I

- Considering phosphor screen as M slabs of thin thickness Δz , the mean number of random x-ray photons absorbed within m th slab located at $z=z_m$:

$$\Phi_m = \mu_p \Delta z \Phi_0 \exp(-\mu_p z_m)$$

where :

Φ_0 = average (constant) fluence irradiated at screen input

$\Phi_0 \exp(-\mu_p z_m)$ = incident x - ray fluence at layer z_m

$\mu_p \Delta z$ = probability of absorption at layer z_m

Input irradiance distribution II

- Each photon absorption of Φ_m creates a flash of optical low energy lights g .

Therefore, average optical photon (Energy) density per unit area of layer z_m is:

$$\langle E_m(r) \rangle = g \Phi_m$$

Methods for Measuring Noise Power Spectra (Wiener)

- Using Fourier Transform of Autocovariance (Autocorrelation) function of the photon Fluctuation

Recall: $\langle E_m(r) \rangle = g \Phi_m$

- The Autocorrelation function of the photon distribution is:

$$R_{E_m}(L) = \langle E_m(r) E_m(r+L) \rangle = g^2 [\Phi_m \delta(L) + \Phi_m^2]$$

filtering process I

- Spreading of light from a point source (where x-ray is absorbed in phosphor) is described by Point Spread Function $P_m(r)$
- Therefore the film exposure is defined by random process:

$$E_m^f(r) = E_m(r) * p_m(r)$$

The Autocorrelation function of filtered photon distribution is:

$$R_{E_m^f}(L) = \langle E_m^f(r) E_m^f(r+L) \rangle = R_{E_m}(L) * p_m(L) \bullet p_m(L) =$$

$$g^2 \Phi_m p_m(L) \bullet p_m(r) + g^2 \Phi_m^2$$

Because (see section 3.5.1): $R_{out}(\tau) = p(\tau) \bullet p(\tau) * R_{in}(\tau)$

filtering process II

Recall: $R_{E^f_m}(L) = g^2 \Phi_m p_m(L) \bullet p_m(r) + g^2 \Phi_m^2$
 $g^2 \Phi_m^2 = \text{Square of mean exposure}$

Autocorrelation function of **exposure fluctuation** is:

$$R_{\Delta E^f_m}(L) = g^2 \Phi_m p_m(L) \bullet p_m(r)$$

Wiener Spectrum is given by FT of the fluctuation $R_{\Delta E^f_m}(L)$:

$$S_{\Delta E^f_m}(u) = g^2 \Phi_m |P_m(u)|^2$$

Where $P_m(u)$ is FT of PSF; $p_m(r)$, the optical transfer function of m th layer.

Autocorrelation of the Fluctuation and Wiener of whole Screen of thickness $M\Delta z=ds$

$$\mathbf{R}_{\Delta E}(L) = \int_0^{ds} g(z)^2 \Phi'(z) [p_z(L) \bullet p_z(L)] dz$$

$$S_{\Delta E}(u) = \int_0^{ds} g(z)^2 \Phi'(z) |P_z(u)|^2 dz$$

where

$$\Phi' = \Phi_0 \mu_p \exp(-\mu_p z)$$

Measurable density fluctuation

- Optical density fluctuation ΔD is related to exposure fluctuation ΔE by:

$$\Delta D = 0.434\gamma\Delta E/E_{avg}$$

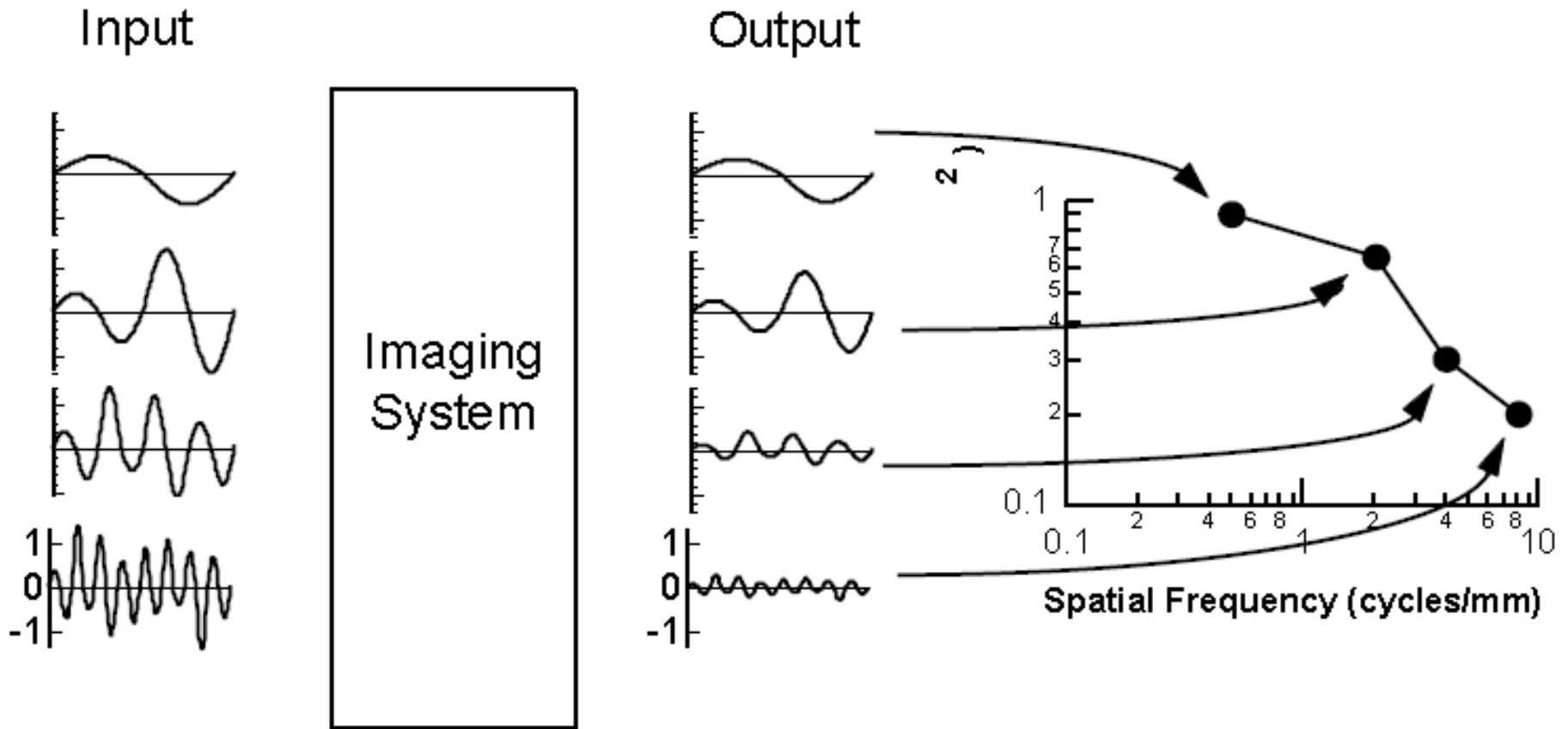
Therefore Wiener spectrum of density fluctuation is:

$$S_{\Delta D}(u) = \left[\frac{0.434\gamma}{E_{avg}} \right]^2 S_{\Delta E}(u)$$

Since the Average exposure is given by: $E_{avg} = \int_0^{ds} g(z)\Phi'(z)dz$

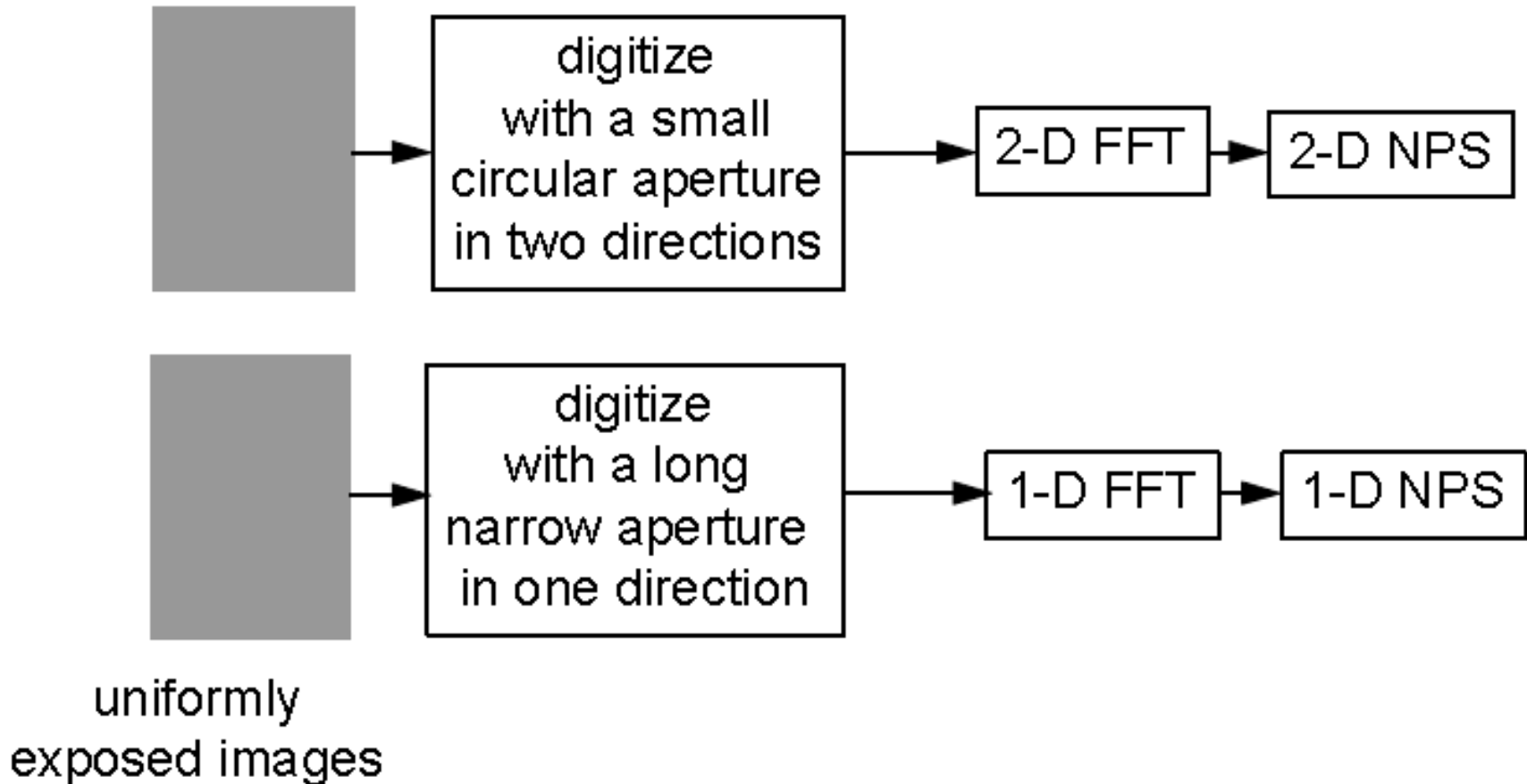
$$S_{\Delta D}(u) = \frac{(0.434\gamma)^2}{\left(\int_0^{ds} g(z)\Phi'(z)dz\right)^2} \int_0^{ds} g^2(z)\Phi'(z)|P_z(u)|^2 dz$$

Measuring NPS (conceptually)



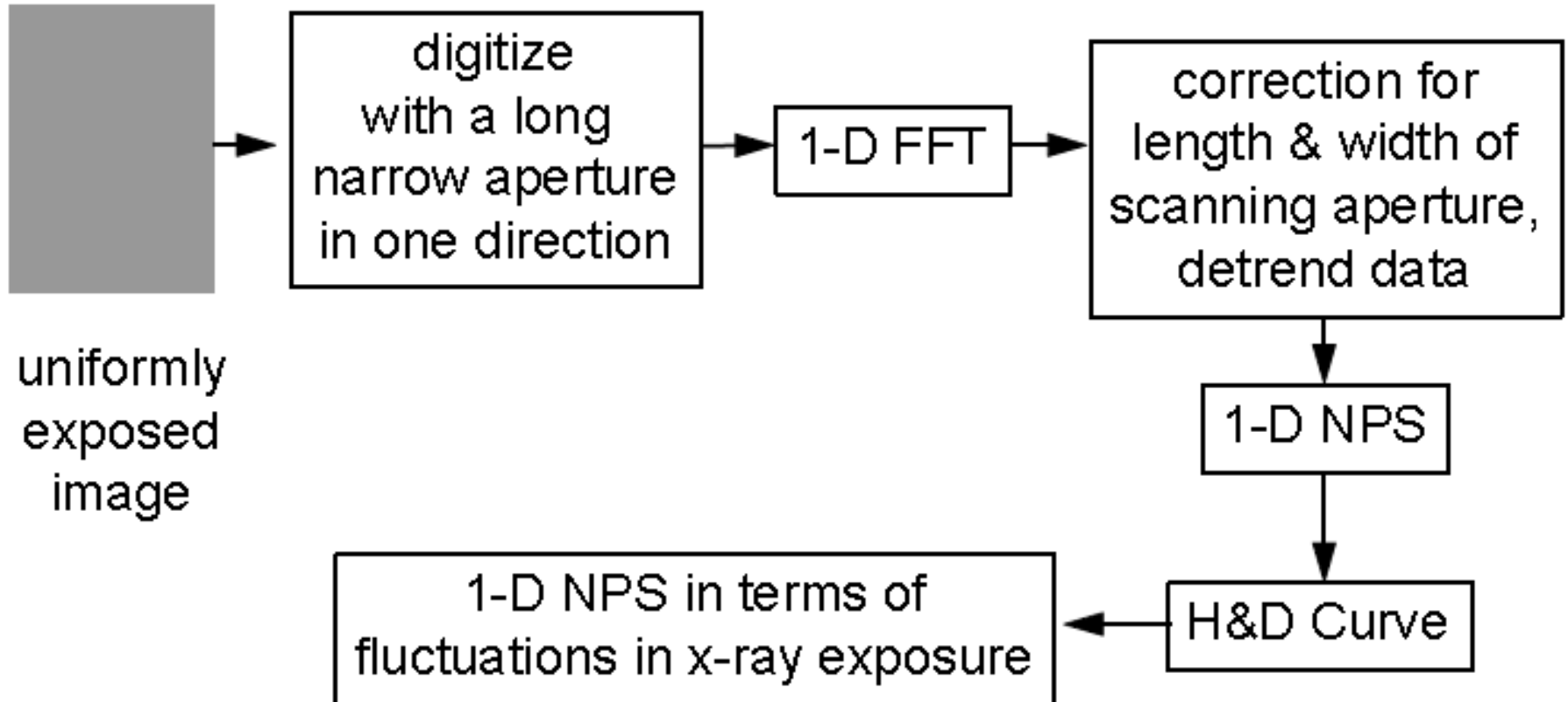
Measure change in the variation in the amplitude of sine waves with noise (Variable signal)

Measuring NPS (theoretically)

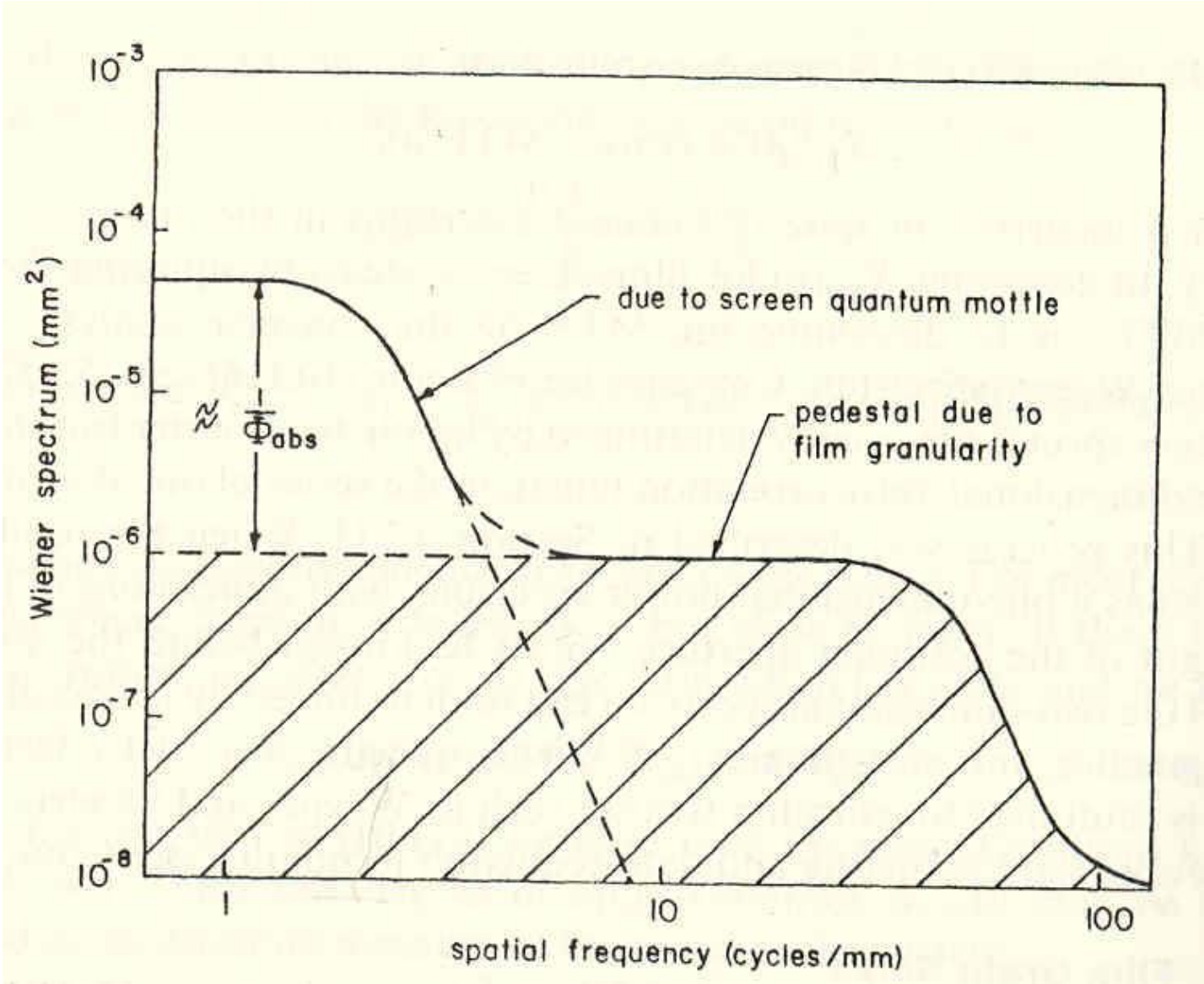


A uniform x-ray exposure contains noise at all spatial frequencies

Measuring NPS (experimentally)

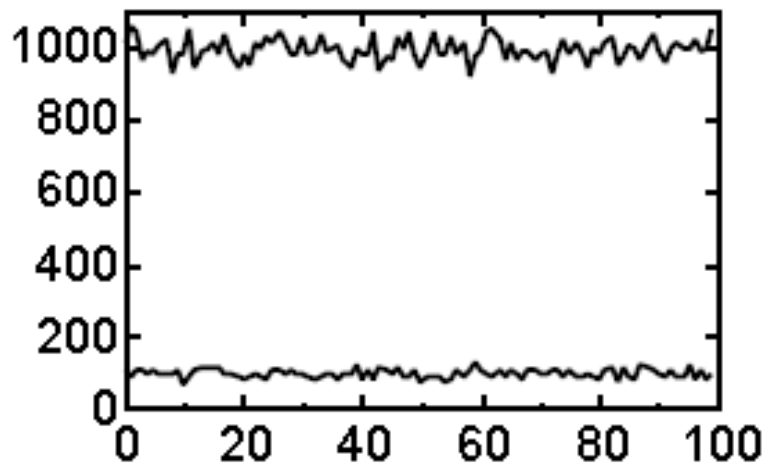


Typical NPS of Film-Screen system



Paradox

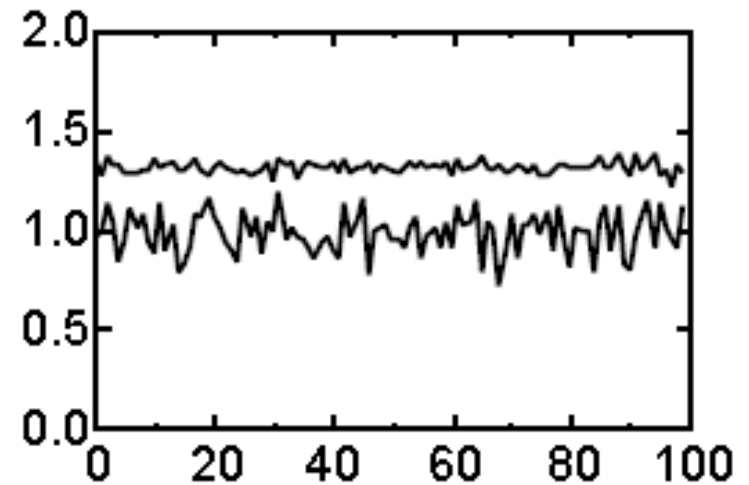
Linear Conversion
(Digital Detector)



Pixel Number

noise increases with exposure

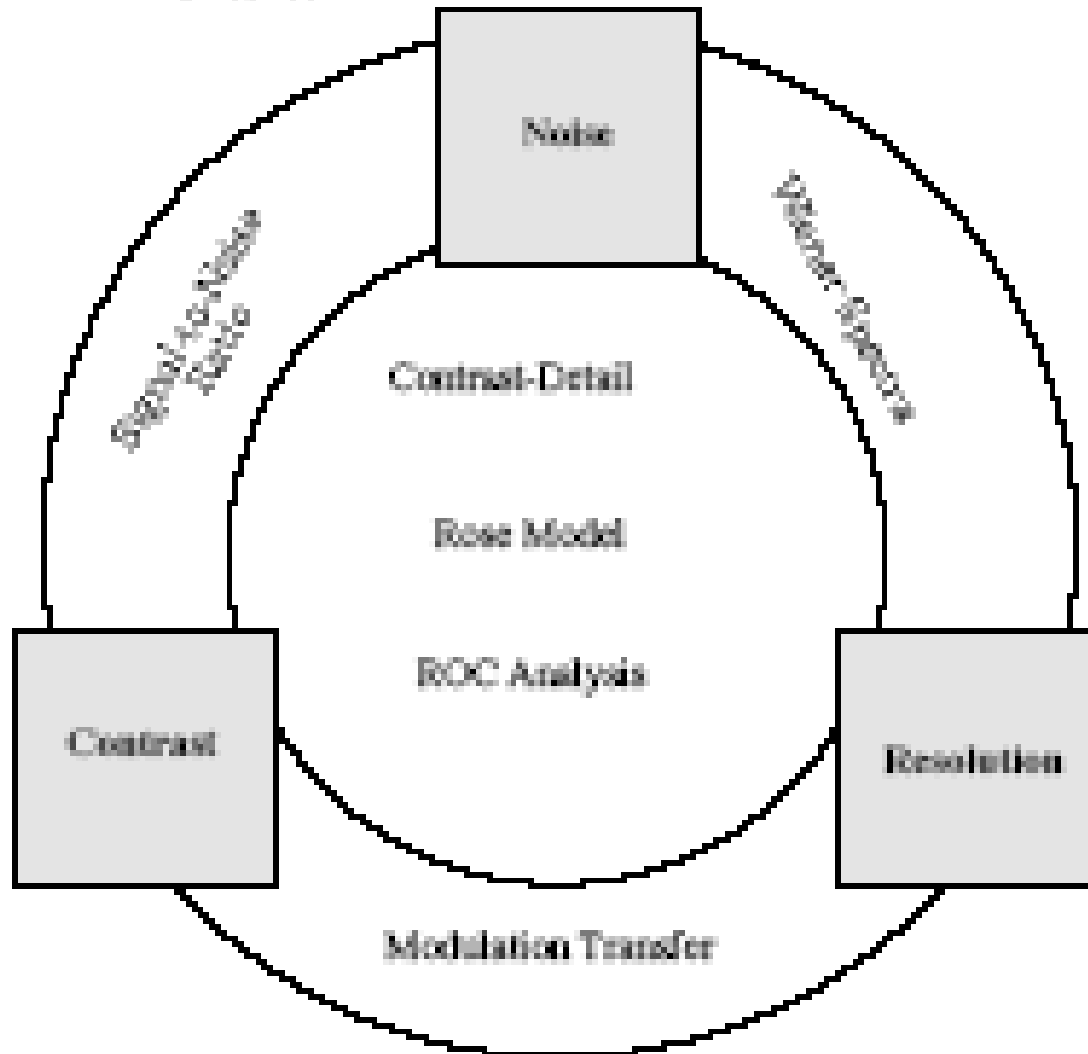
Logarithmic Conversion
(Screen-Film System)



Pixel Number

- noise decreases with exposure

Imaging concepts used to model imaging system performance.



Optimum Filters for noise

- It depends on:
 - The purpose of system usage
 - The nature of input to the filter
 - The definition of the best
- Two main filters:
 - Wiener filter
 - Matched filter

Wiener filter

- The aim is to extract a signal from additive, signal-independent noise
- Spectral information or autocorrelation of signal or/and noise is known
- The signal is tried to be estimated so that the difference between this estimation and the original signal (random process) property is minimized

Wiener filter

- The input to filter is random process $y(t)$ comprising the real signal contaminated by noise:

$$y(t) = S(t) + n(t)$$

An estimated signal $\hat{s}(t)$ is obtained by filtering $y(t)$ with wiener filter $p(t)$:

$$\hat{s}(t) = y(t) * p(t)$$

Then the mean Square error should be minimized by:

$$\langle |\hat{S}(t) - S(t)|^2 \rangle = \text{minimum}$$

Wiener filter

- It can be shown that the $p(t)$ that minimizes $\langle |\hat{S}(t) - S(t)|^2 \rangle$ makes the error $e(t) = \hat{S}(t) - S(t)$ orthogonal to the data $y(t)$.

$$\langle [\hat{S}(t) - S(t)] y(t') \rangle = 0$$

Also signal and noise are uncorrelated:

$$\langle s(t)n(t') \rangle = 0$$

Therefore:

$$\begin{aligned} \langle \{ [s(t)^* p(t)] + [n(t)^* p(t)] - s(t) \} \{ s(t') + n(t') \} \rangle = \\ \int \langle s(t'') s(t') \rangle p(t-t'') dt'' + \int \langle n(t'') n(t') \rangle p(t-t'') dt'' - \langle s(t) s(t') \rangle = 0 \end{aligned}$$

Wiener filter

- Since each term is an autocorrelation function:

$$R_s(t-t') * p(t-t') + R_n(t-t') * p(t-t') - R_s(t-t') = 0$$

Fourier Transform of this equation gives:

$$S_s(u)P(u) + S_n(u)P(u) = S_s(u)$$

Where S_s and S_n are power spectral density of s and n , and $P(u)$ is optimum filter with transfer function given as:

$$P(u) = S_s(u) / [S_s(u) + S_n(u)]$$

This means $P(u)$ should be large for frequencies at which the signal power is larger than noise power, and vice versa

Matched filter

- Wiener filter was optimal for estimation problem based on an unknown random variable as input signal, therefore the estimated filtered output resemble the input.
- Matched filtering is based on a known input signal which should be detected using an optimum filter.
- An optimized filter is the one which maximize SNR of the output. eg;
$$[\text{SNR}(t)]^2 = [s(t)*p(t)]^2 / [n(t)*p(t)]^2 = \text{Maximize}$$