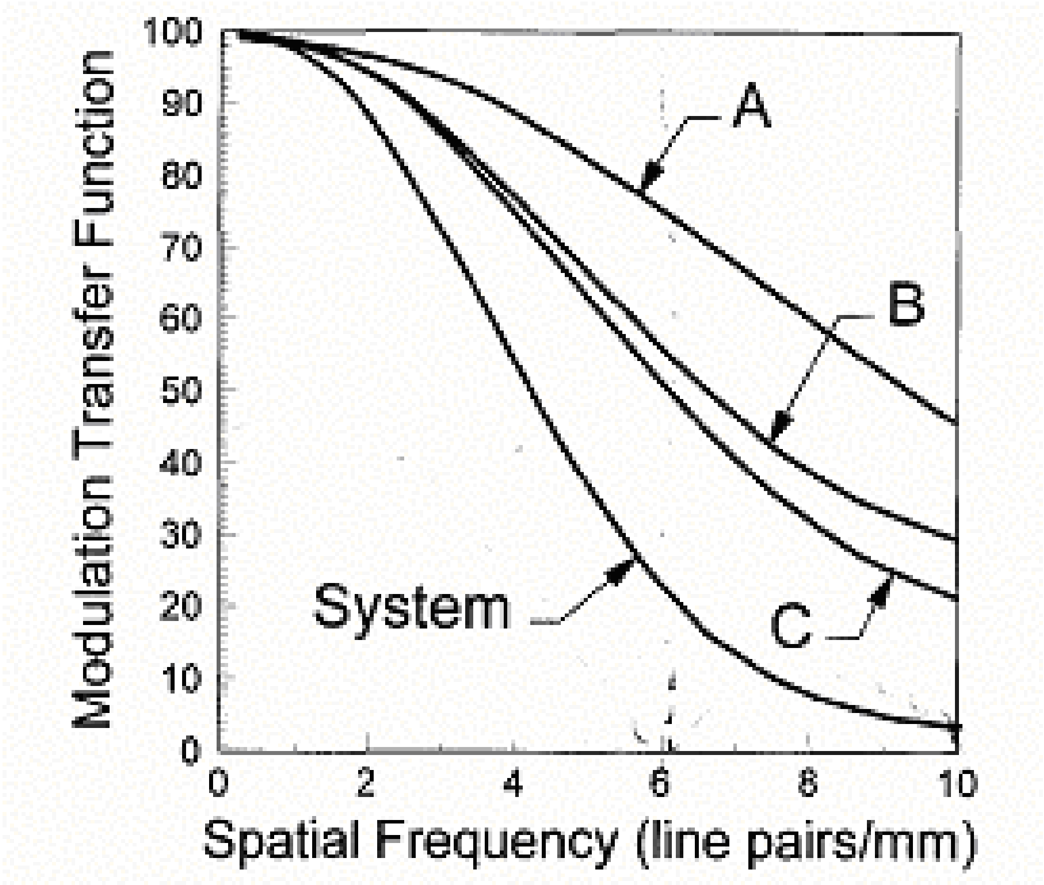
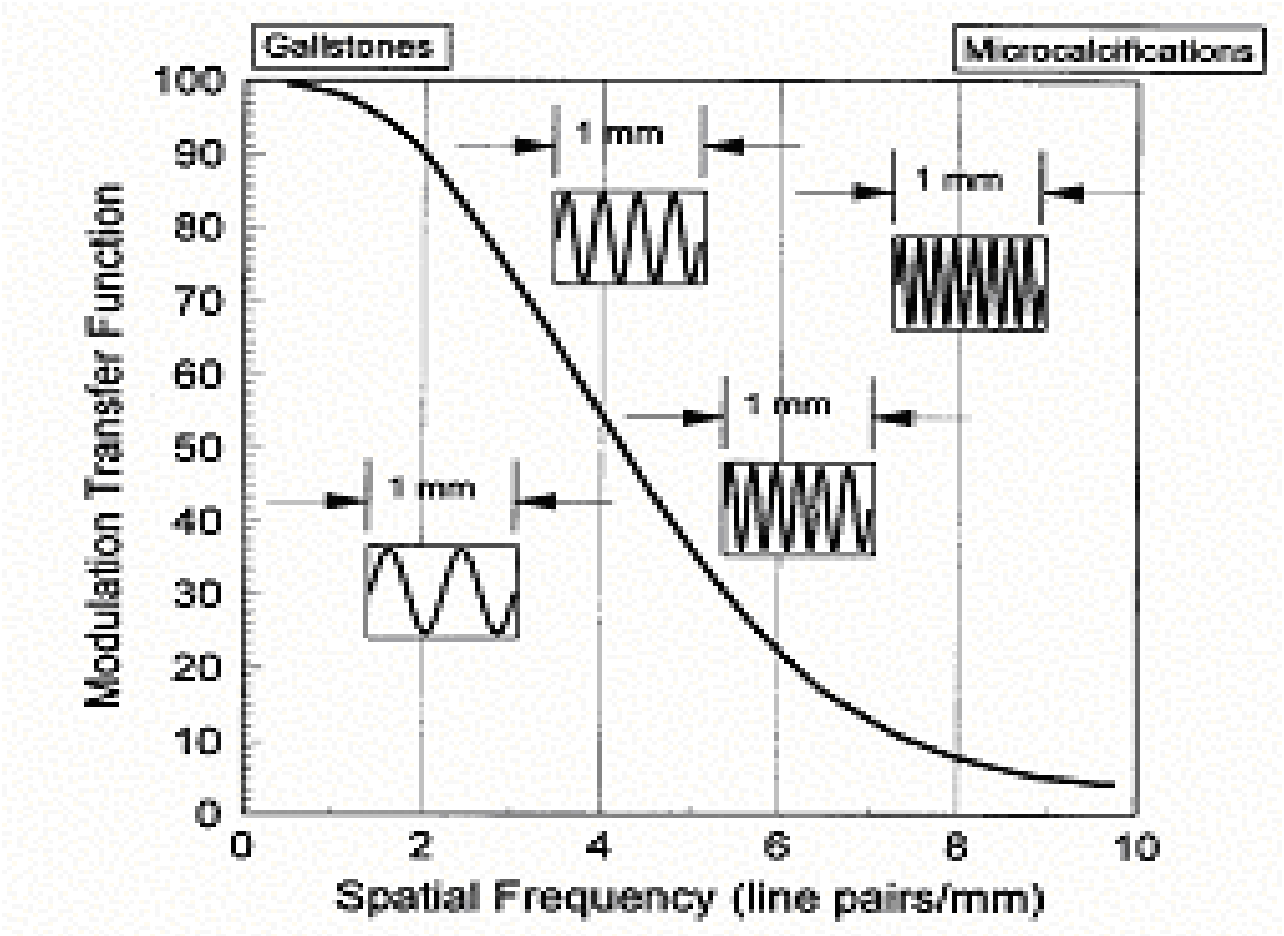
Modulation Transfer Function (MTF)

Graphical description of system resolution showing the quality of object transferred to image.

eg; If %80 of object contrast is present in image, then MTP is 0.8





If object size (eg; The thickness of slit in a lead plate or Line in LSF) is Δ, then spatial frequency is:

Spatial Frequency =1/(2Δ)

- **Frequency and space (time) are convertible to each other. Therefore, taking Fourier Transform of LSF we get MTF.**

h(x,y)=LSF

$$H(U,V) = \int\int_{-\infty}^{+\infty} h(x,y) \ \exp[-2\pi i(ux+vy)] \ dx \ dy$$

$$\boldsymbol{MTF(U,V) = |H(U,V)|}$$

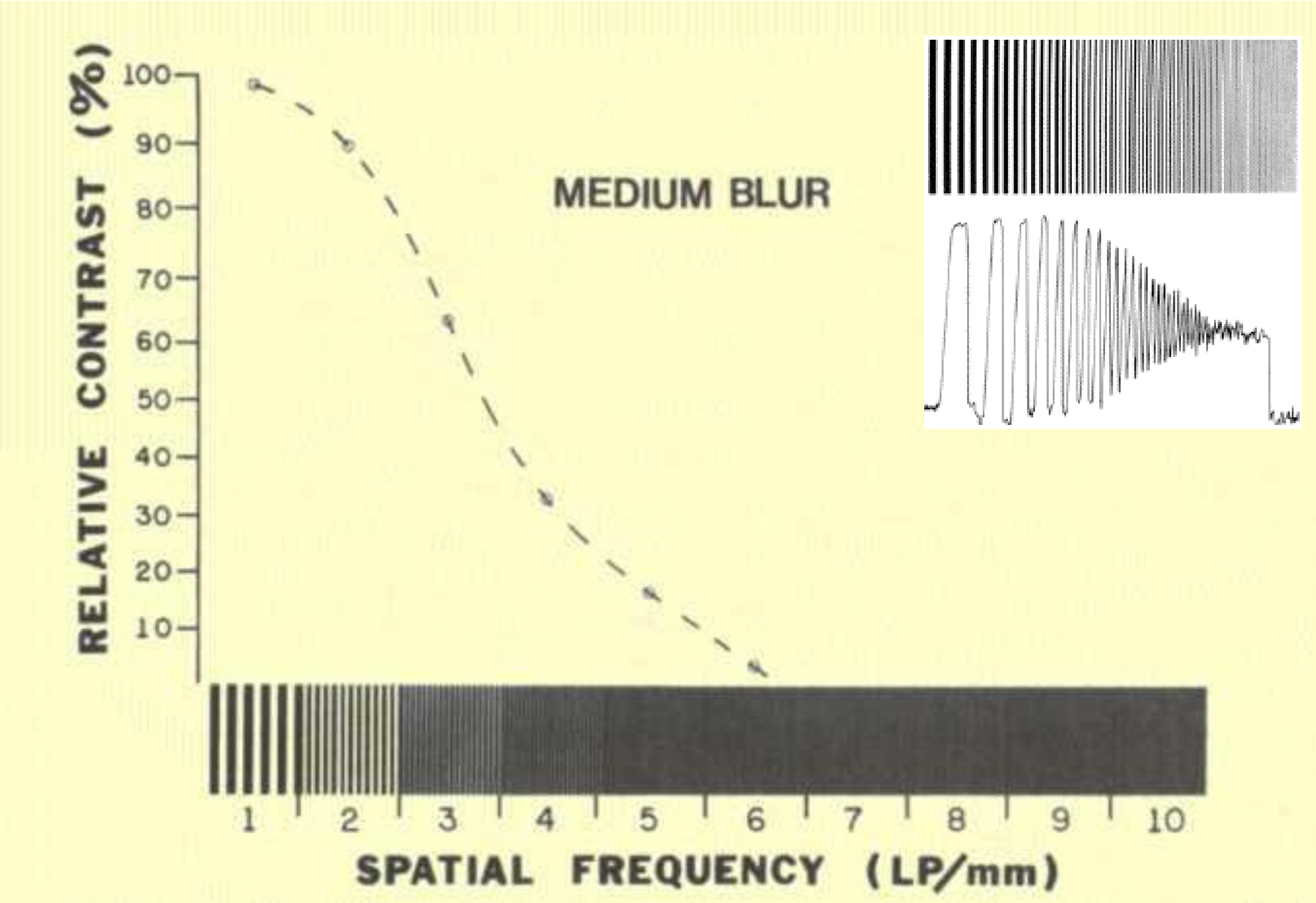
- **An object can be represented by its frequency components (Sinusoidal signals) using FT.**

- **Then comparing these information with the MTF of system, we can evaluate the system to see if it can transfer the essential details (eg; Whether it can reconstruct the degraded information lost by Sampling)**

- **FT of Noise is also useful to evaluate the system for transferring or eliminating noise information through the system.**

- **FT of noise (Power spectrum or Wiener spectrum) is compared with the MTF of the system.**

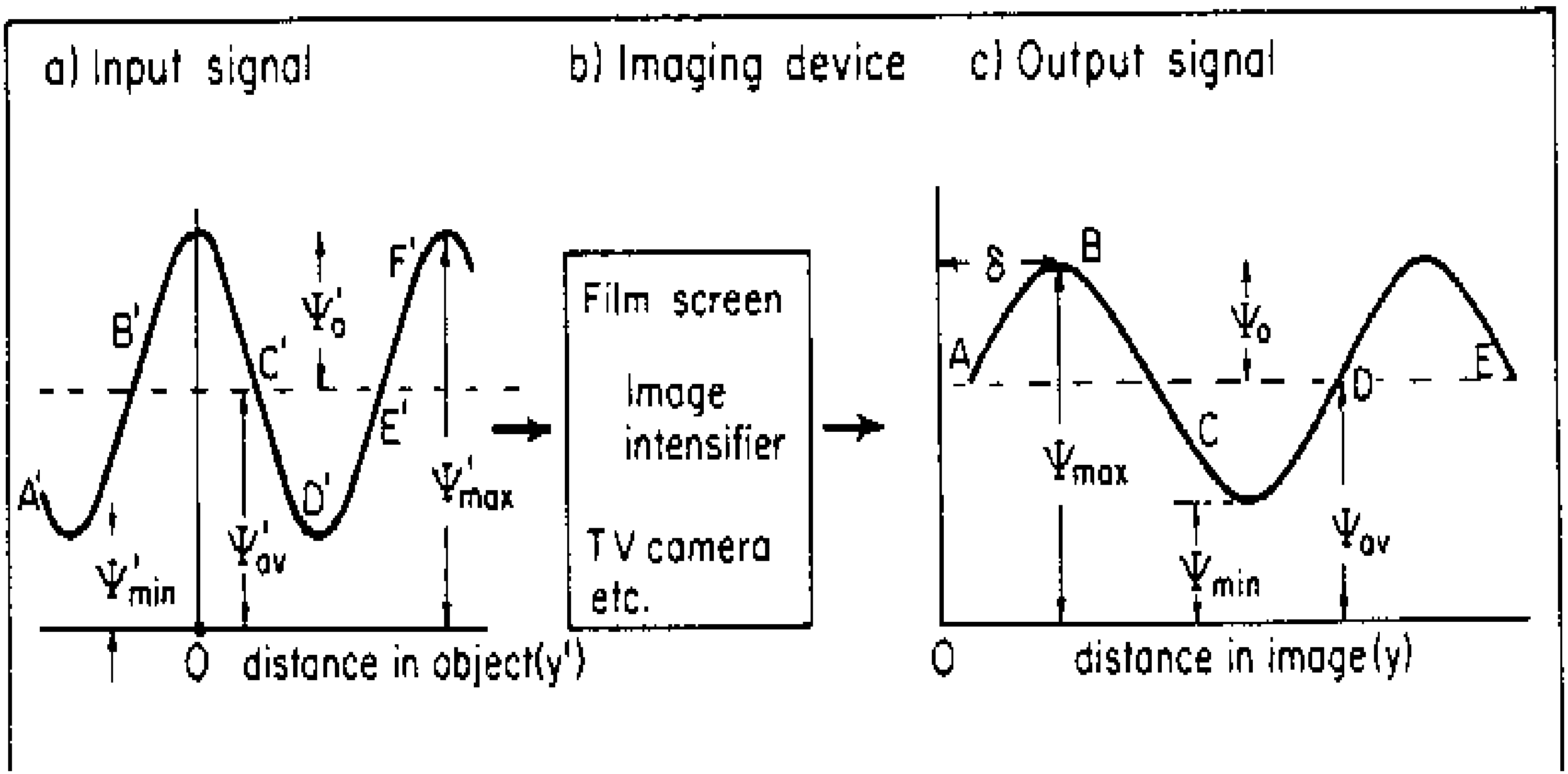
Using Square (Sinusoidal) test Pattern for MTF



MTF measurement

- A sinusoidal test phantom (a series of slits or square shape holes) is fixed in distance F from Source and d from Film. M=(F+d)/F

Then, the image of slits at various size are obtained.



MTF measurement

- **Modulation (Contrast) of object is obtained for a point source as *M'***

$$M' = \frac{T_o}{T_{av}}$$

- ***Where:***
- ***T_o = density changes***
- ***T_{av} = Average density on film***
- ***T_o^+ = Maximum density***
- ***T_o^- = Minimum density***

MTF measurement

- **Modulation (Contrast) of image is obtained using the actual Focal Spot *M***

$$M = \frac{T_o \quad H(f)}{T_{av}} \Rightarrow \qquad MTF = \frac{M}{M'} = H(f)$$

- ***H(f) is less than 1, and It can be shown that H(f) is equivalent to FT of Focal Spot (PSF) with an addition of Magnification factor M.***

- ***Therefore for calculation of MTF, first FT of Focal Spot should be obtained, then its Frequency is modified (decreased) by M.***

$$f = f_0 / M$$

Experimental measurement of MTF

- **Input Modulation is:**

$$M'(f) = \frac{A'_{\max} - A'_{\min}}{A'_{\max} + A'_{\min}}$$

A = Amplitude

- **Output Modulation is:**

$$M(f) = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

Usualy Output Modulation is smaller than Input Modulation

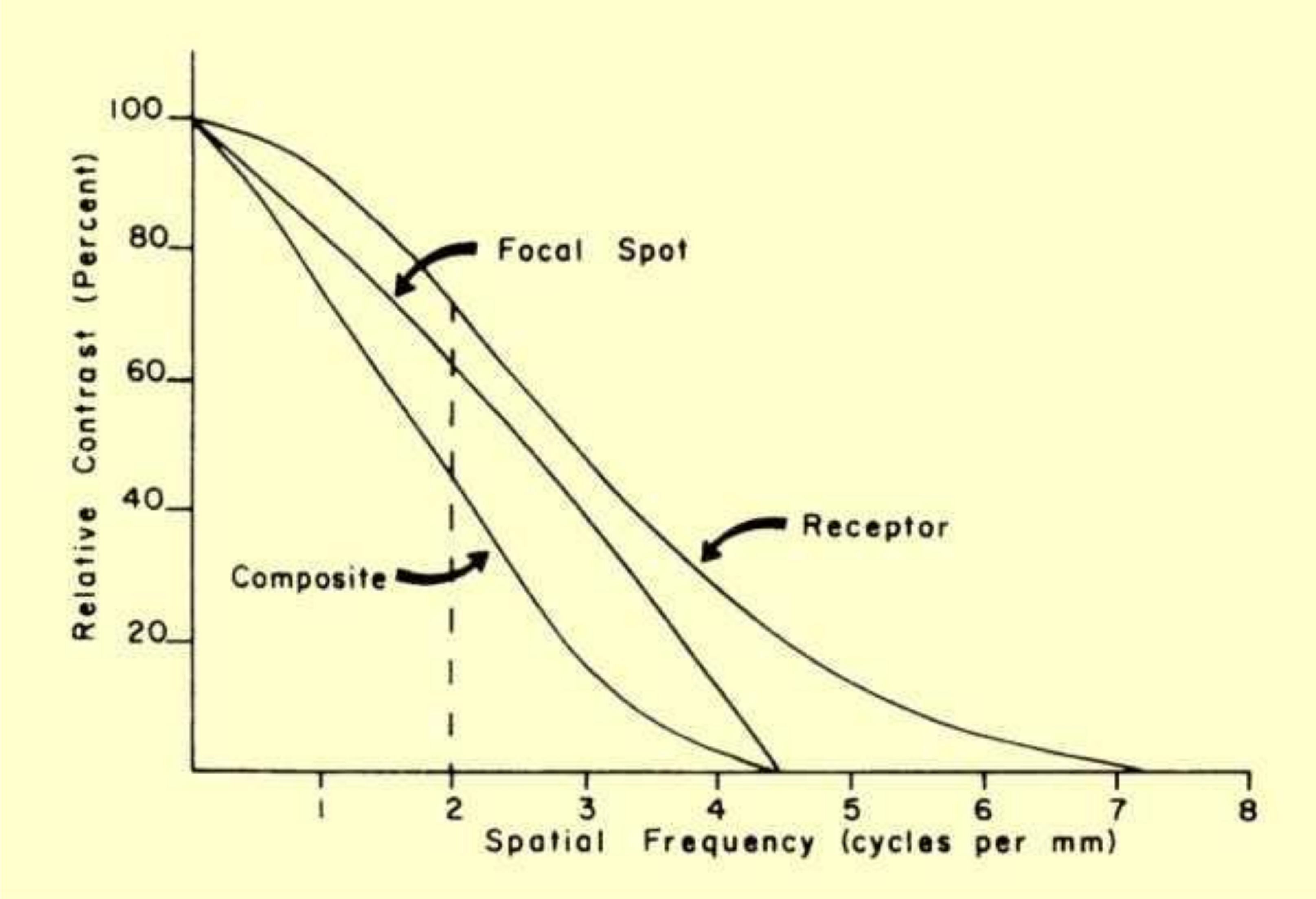
$$Modulation\ Transfer\ (MT) = \frac{Modulation\ of\ output\ signal}{Modulation\ of\ input\ signal} = \frac{M}{M'}$$

Since the Modulation of both Input and Output are similar at frequency of 0, therefore MTF is 1 in zero frequency.

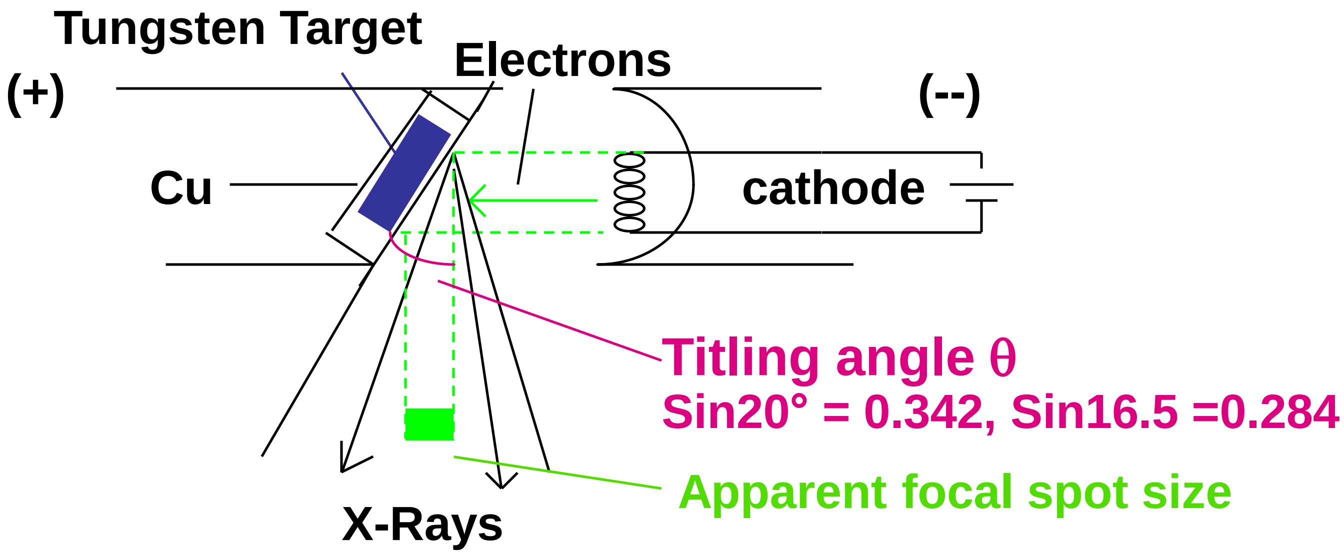
In addition the MTF of Input is close to 1 at all frequencies. therefore, MTF of Output can be normalized and written as:

$$\mathrm{MTF} = \frac{M(\mathrm{f})}{M'(\mathrm{f})} \times \frac{M'(0)}{M(0)} = \frac{M(\mathrm{f})}{M(0)}$$

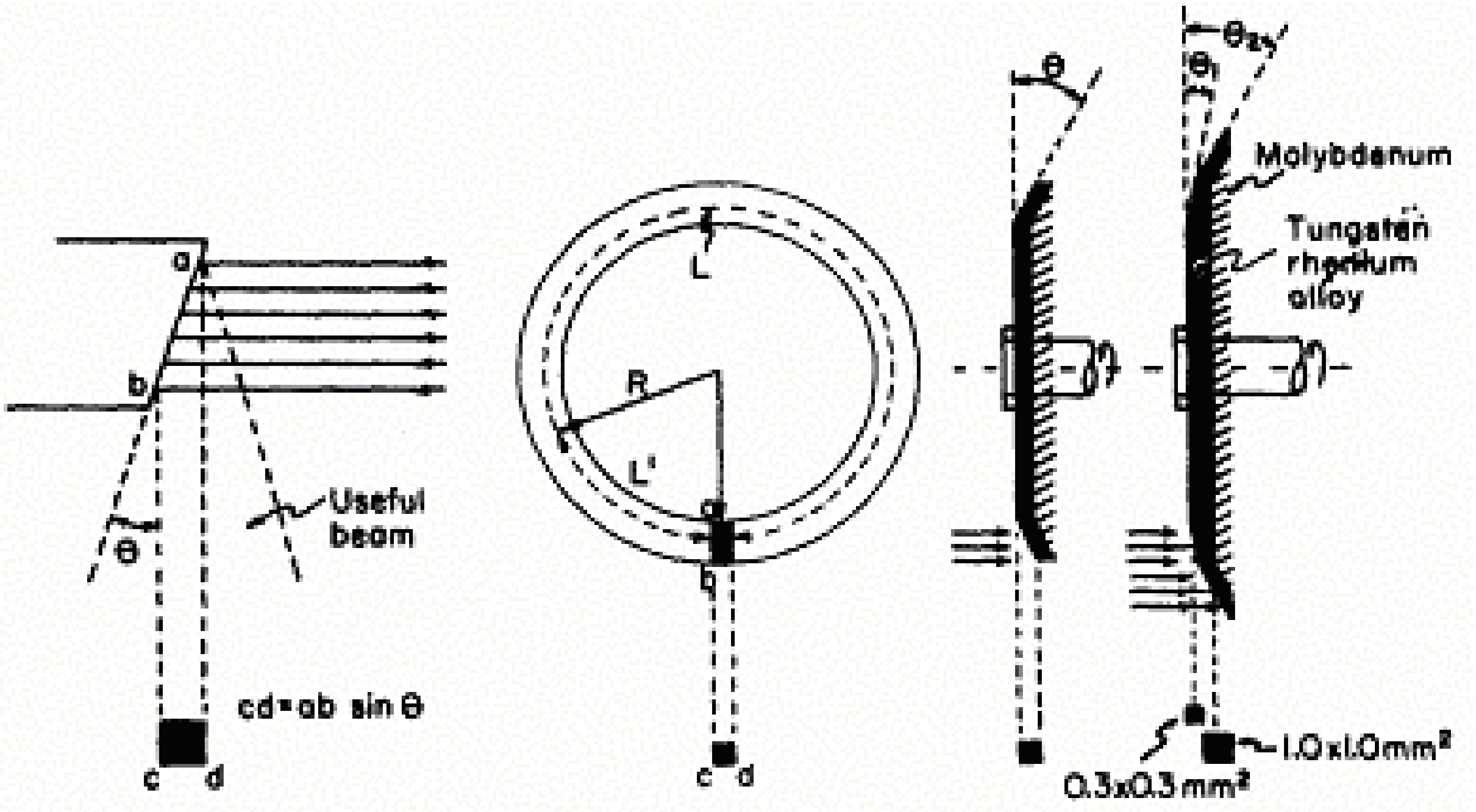
Expected MTP of x-ray system components

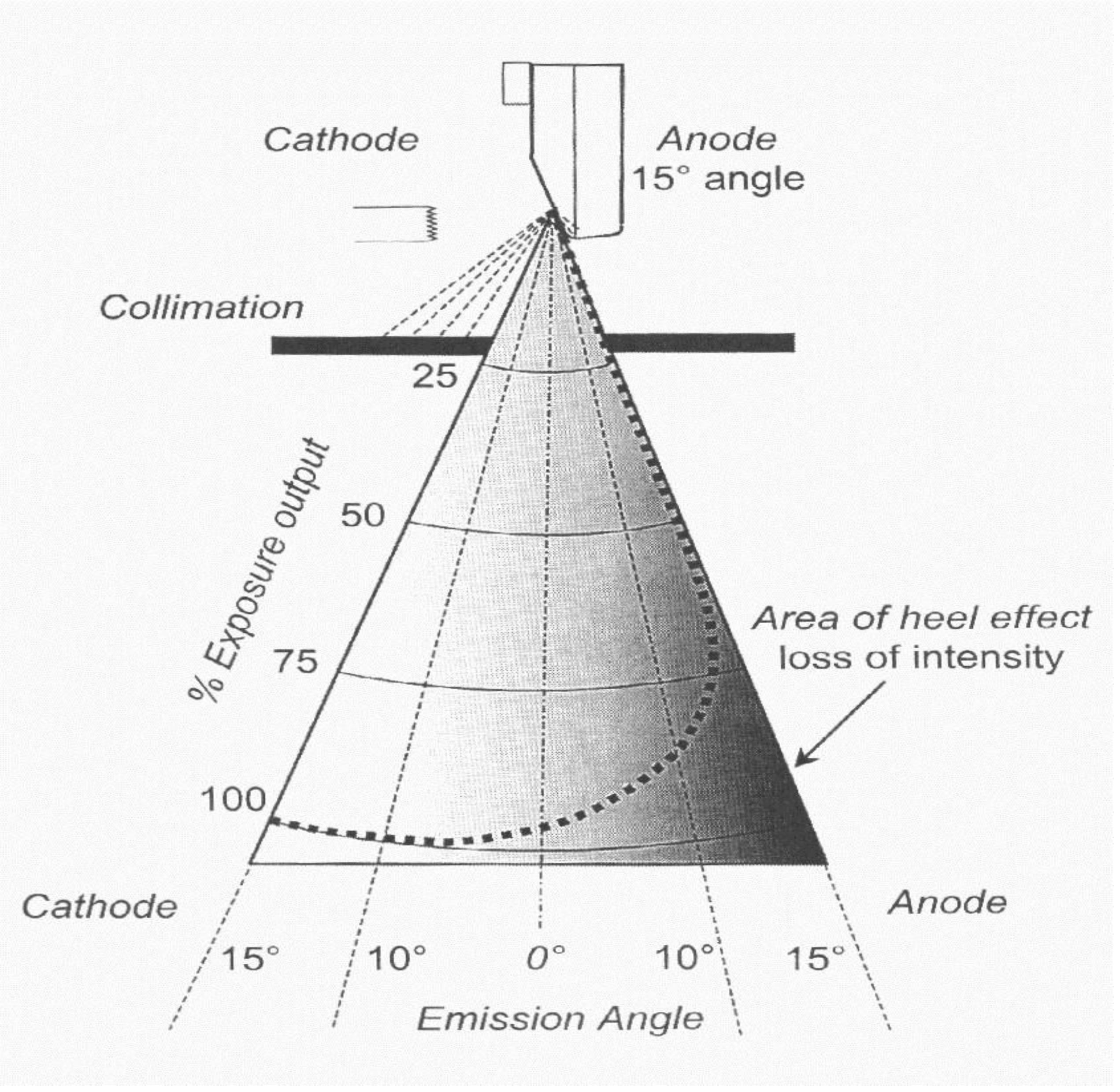


MTF of Focal Spot

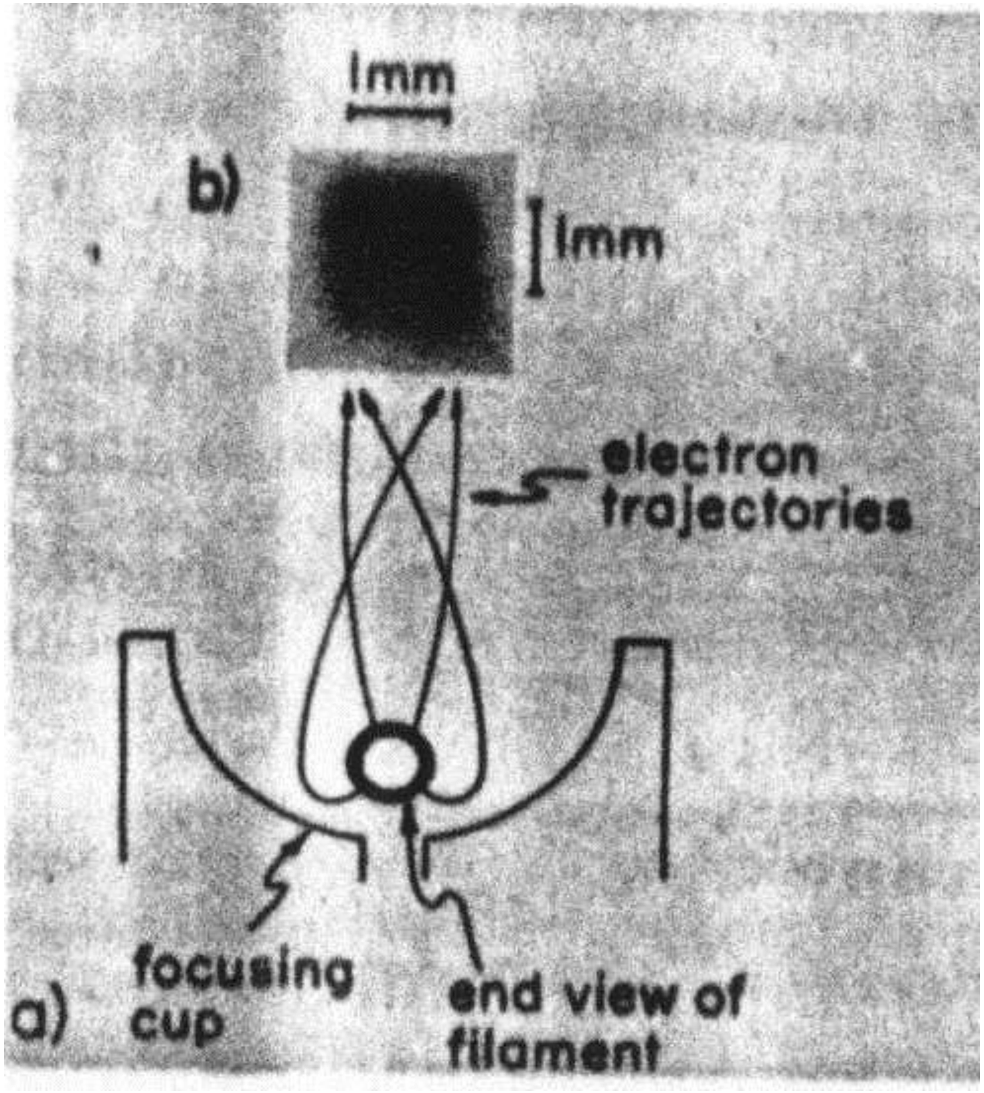

Tungsten Target
Electrons
(+)
(--)
Cu
cathode
Titling angle θ
Sin20° = 0.342, Sin16.5 =0.284
Apparent focal spot size
X-Rays

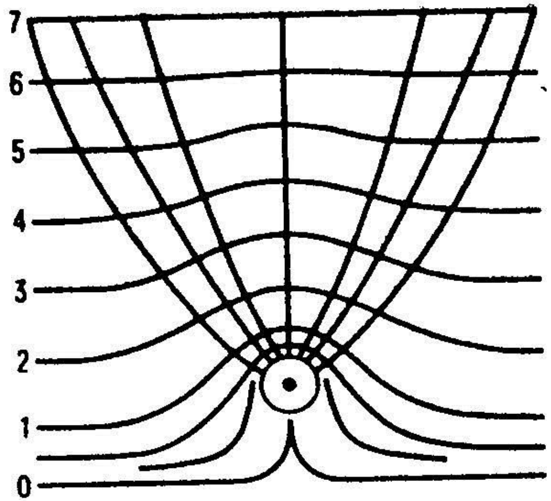
Focal Spot


a
b
θ
Useful beam
cd = ab sin θ
c
d
L
R
L'
a
b
c
d
θ
θ2
θ1
Molybdenum
Tungsten rhenium alloy
1.0x1.0mm²
0.3x0.3 mm²

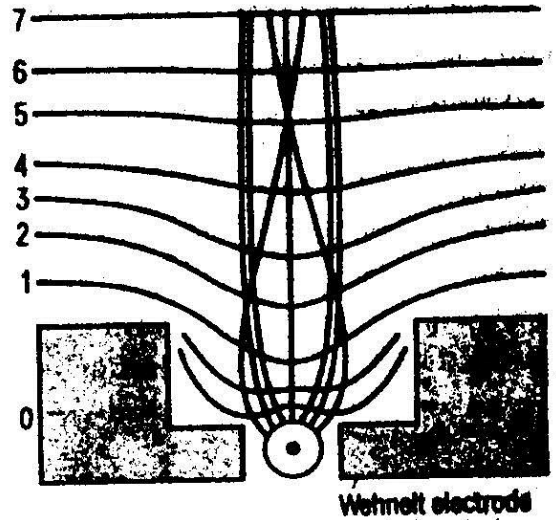

Cathode
Anode
15° angle
Collimation
25
50
75
100
% Exposure output
Area of heel effect
loss of intensity
Cathode
Anode
15°
10°
0°
10°
15°
Emission Angle

Focal Spot MTF

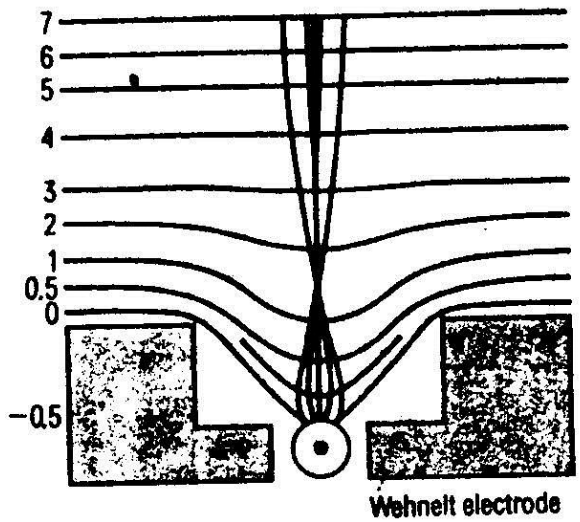


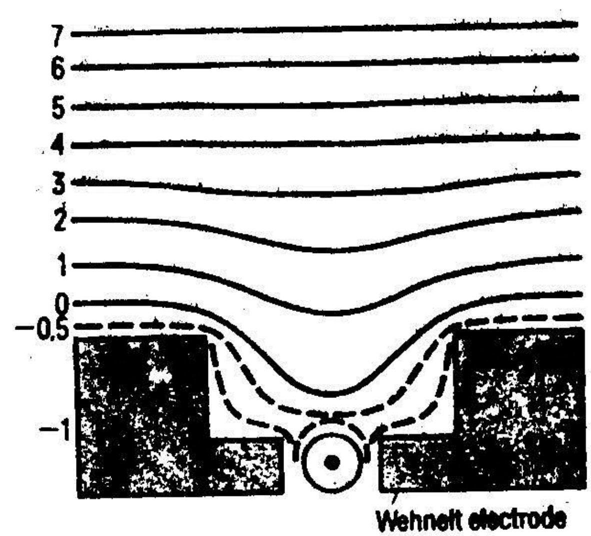


a) Cylindrical filament opposite plane anode

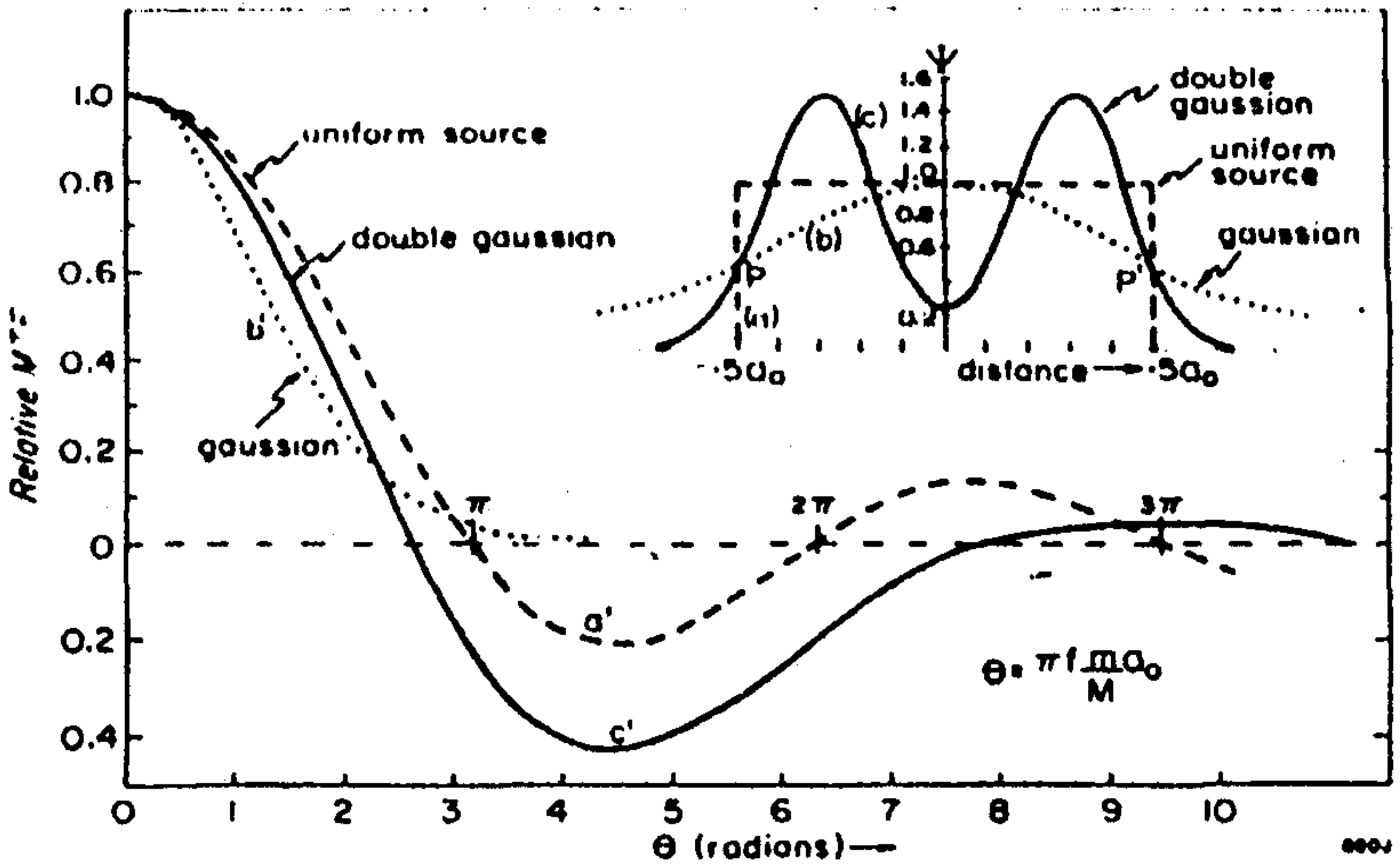


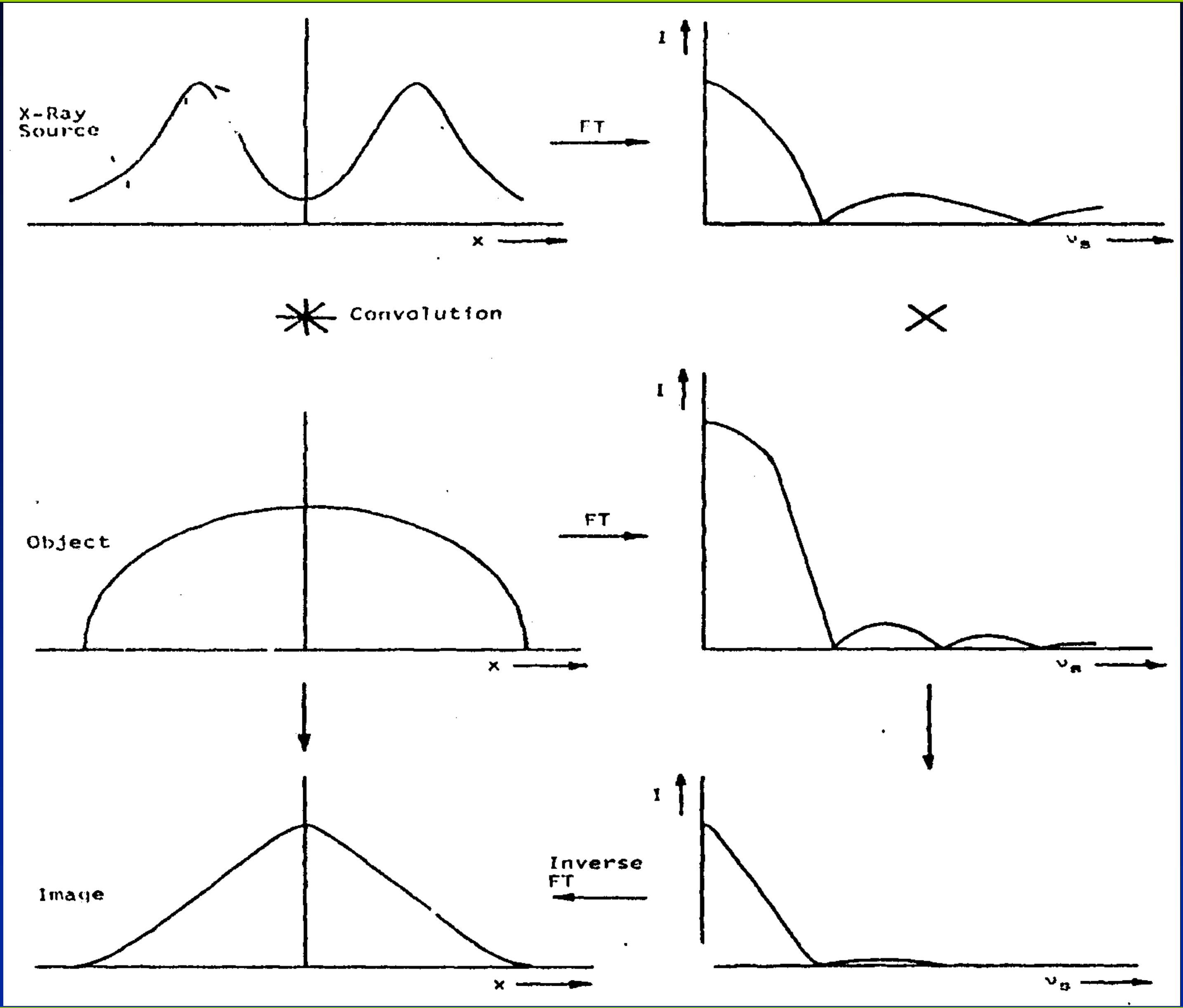
b) Focussing effect of the Wehnelt electrode





MTF of various shape of Focal Spot




X-Ray
Source
FT
I
x
ν_s
Convolution
I
Object
FT
x
ν_s
I
Image
Inverse
FT
x
ν_s

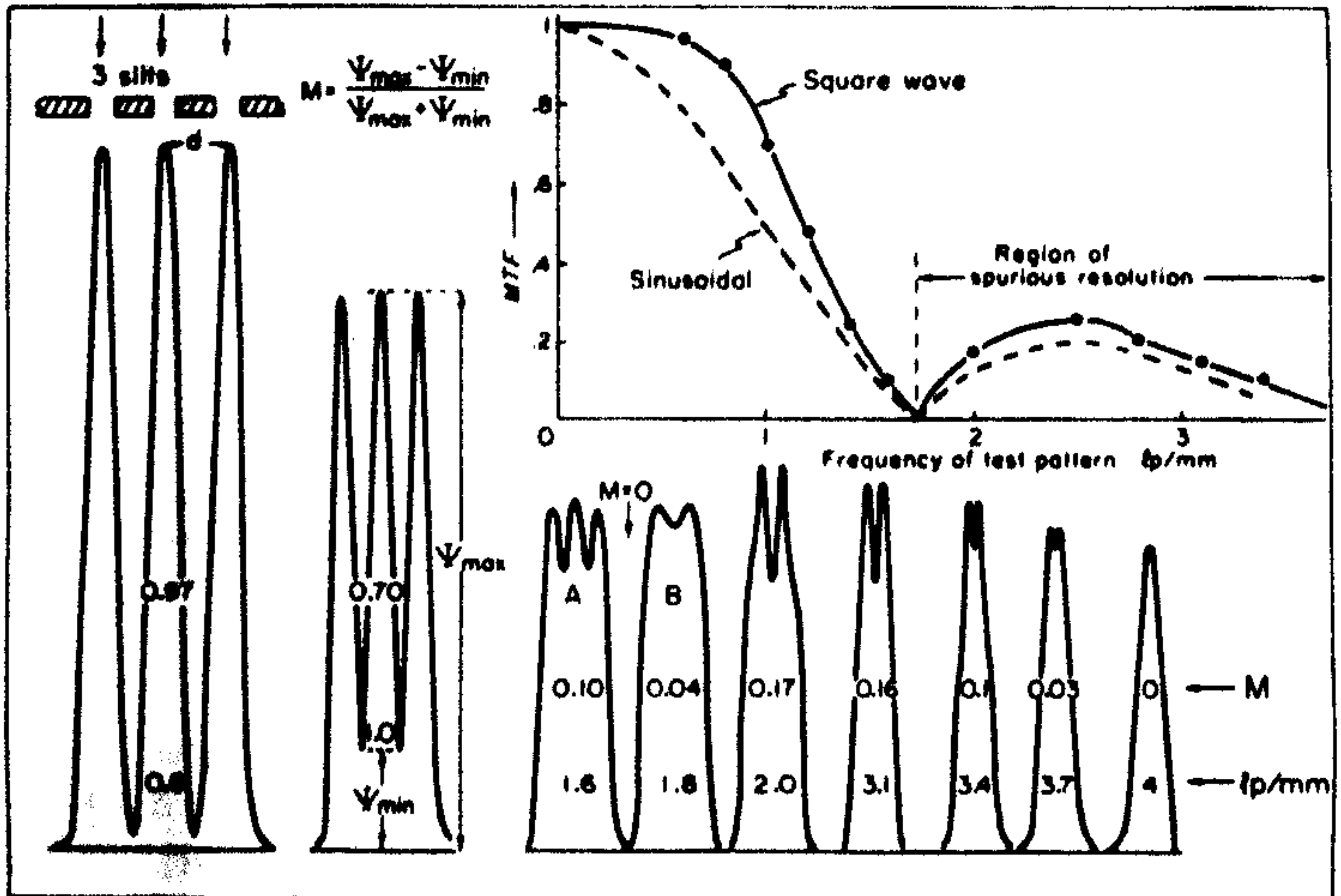
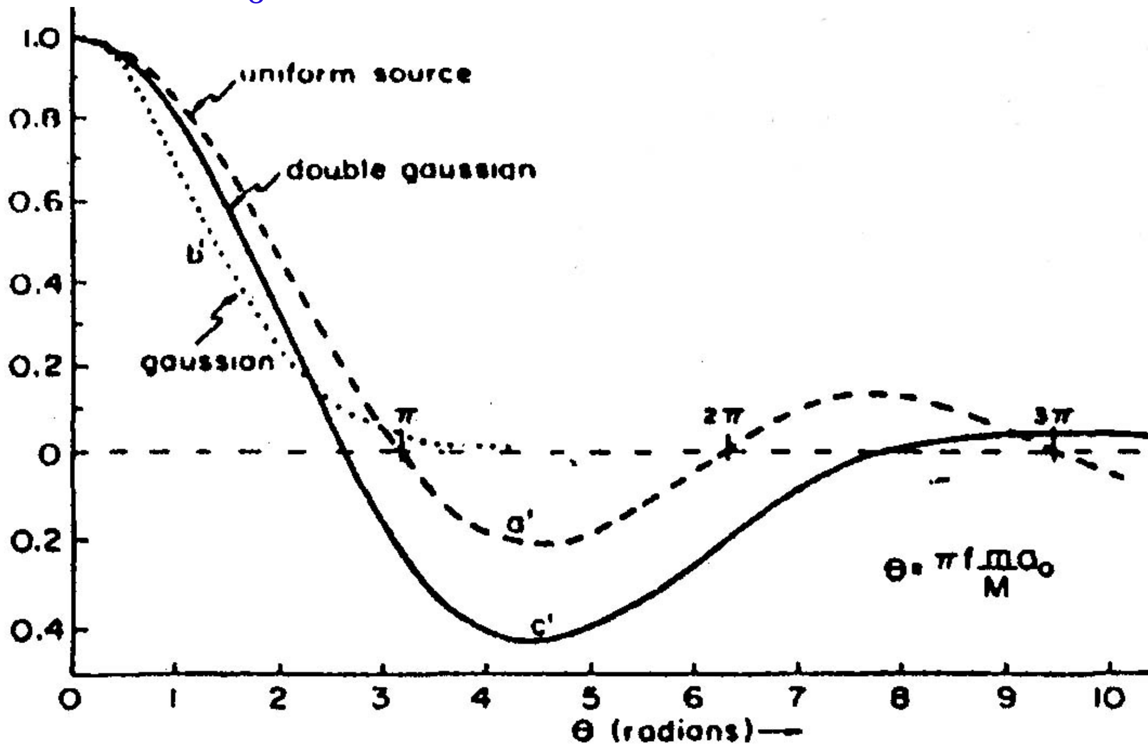


Figure 16-21. Fluence through a square wave test pattern for frequencies of 0.6, 1.0, 1.6, 1.8, 2.0, 3.1, 3.4, 3.7, and 4.0 lp/mm. The magnification was 1.7 so that, for example, the distance d = 1.7(1/0.6) = 2.83 mm.

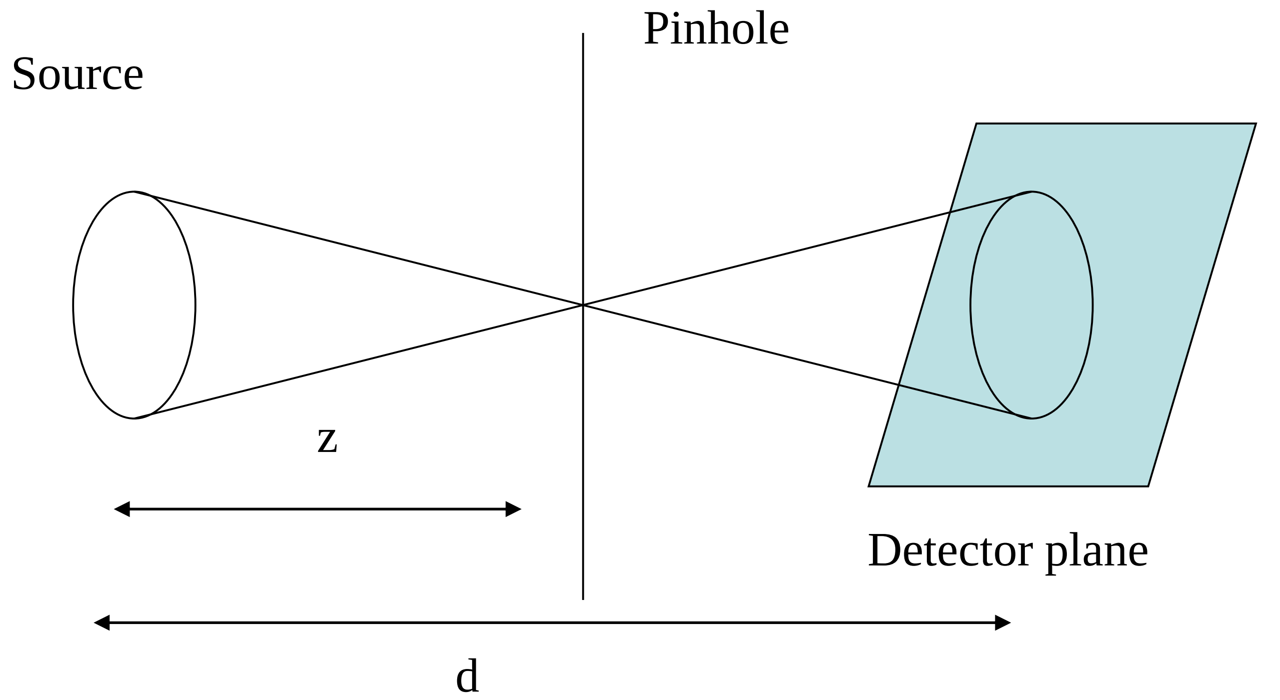
Focal Spots MTF depend on the shape and size of focal spot and magnification of image.For example; MTF of Square or Double pick focal spot is:

$$MTF = H(f) = FT(f)_{PSF} = \frac{Sin\,\theta}{\theta} \Leftarrow \theta = \frac{\pi f m a_0}{M}$$

$$MTF = 0 \begin{cases} \theta = \pi, 2\pi, 3\pi, \ldots \Leftrightarrow \text{where } \frac{fma_o}{M} = 1,2,3,\ldots \\ \text{if } M = 2 \longrightarrow fa_o = 1,2,3,\ldots \end{cases}$$



M=Magnification
m = M –1
a_0 = focal spot width
f = frequency of signal



$$M = \frac{d}{z}\, and\ m = \frac{-(d-z)}{z} = 1 - M$$

Finite source

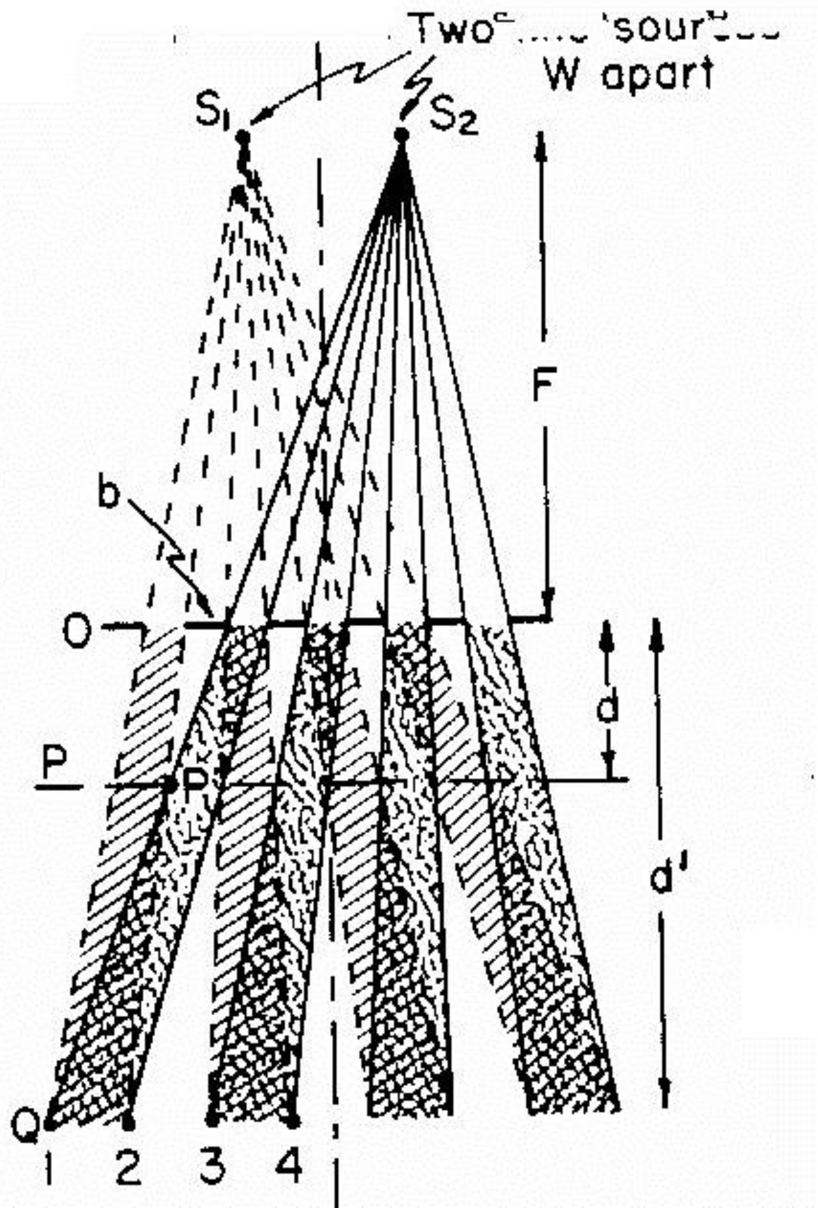
The total detected Intensity (image) of a transparent object (hole) having transmission t(x,y) = exp [-μ (x,y) δ (z - z_o)] imaged by a finite x-ray source, s(x,y) is obtained by convolution process:

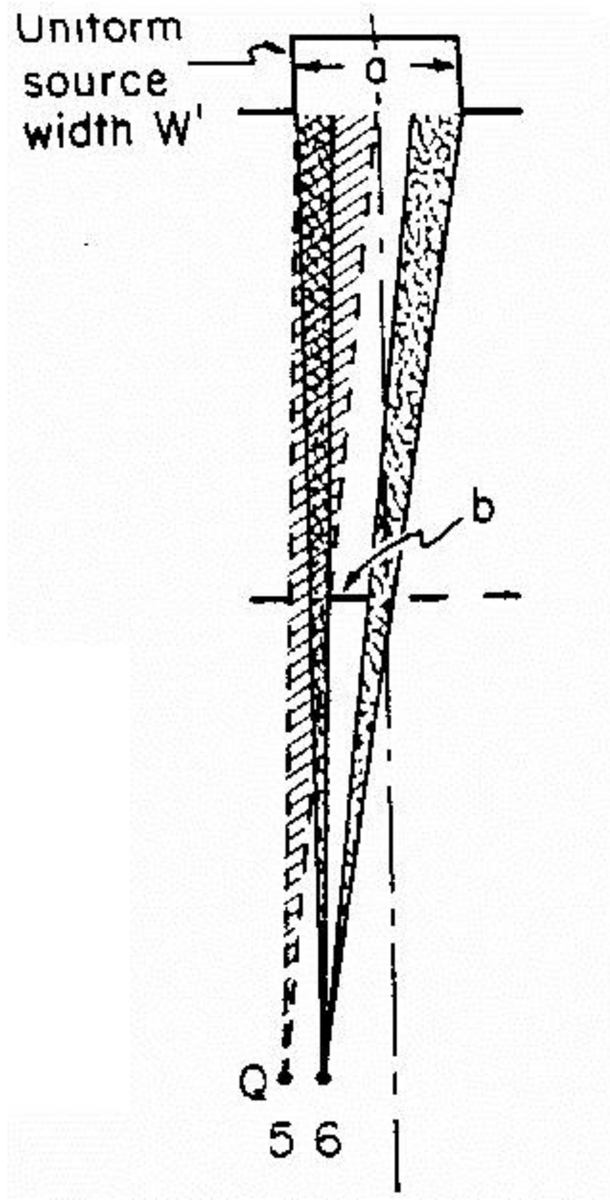
$$I_d(x_d, y_d) = Kt(\frac{x_d}{M}, \frac{y_d}{M}) ** s(\frac{x_d}{m}, \frac{y_d}{m})$$

The detected image will be the convolution of a agnified object and a agnified source.
In frequency domain:

$$I_d(u,v) = KM^2 m^2 T(Mu, Mv) S(mu, mv)$$

Two sources Radiating a Test Pattern

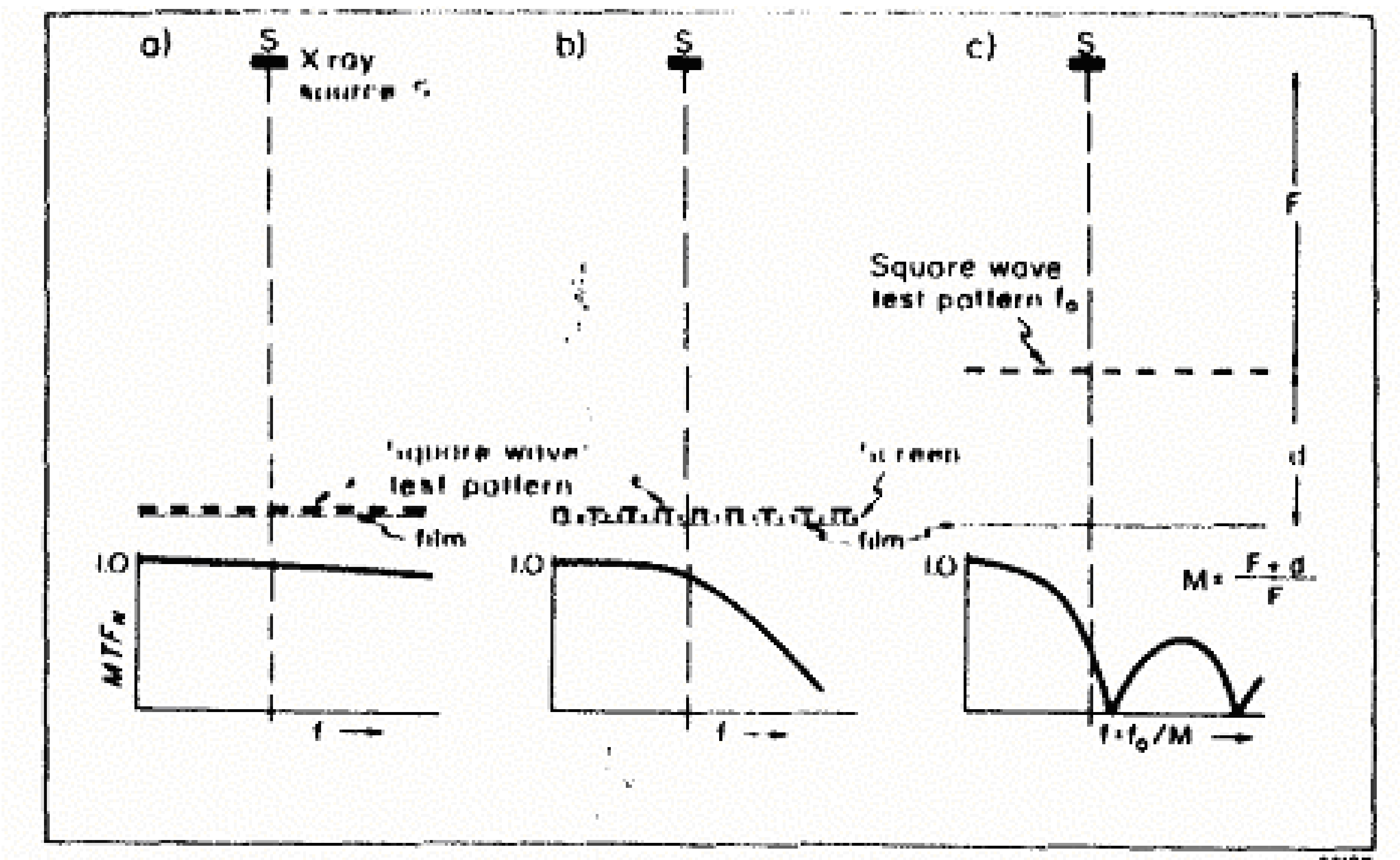




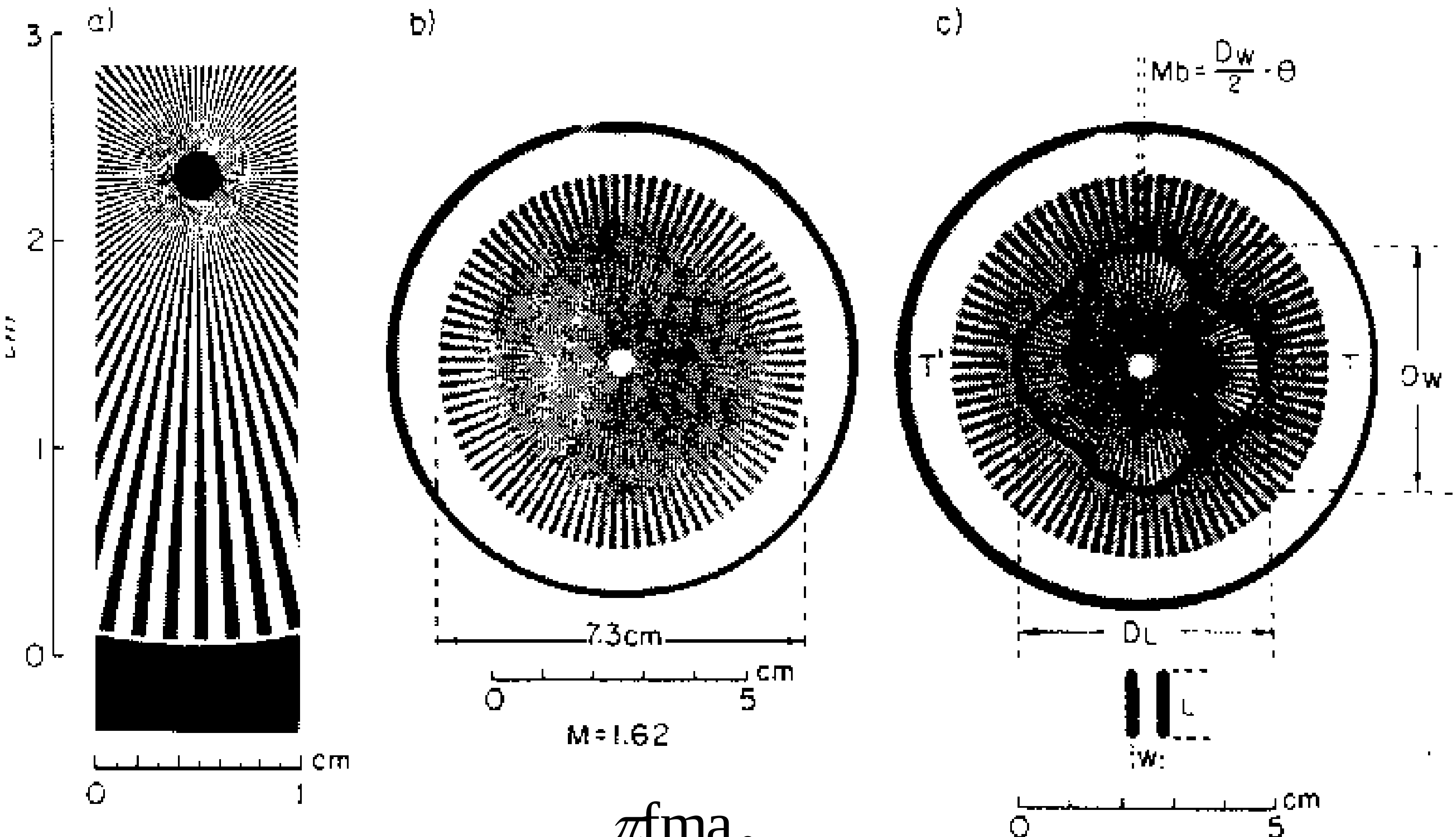
MTF of different System components

Square wave test pattern a) in contact with Film
b) in contact with Screen
c) same distance from film

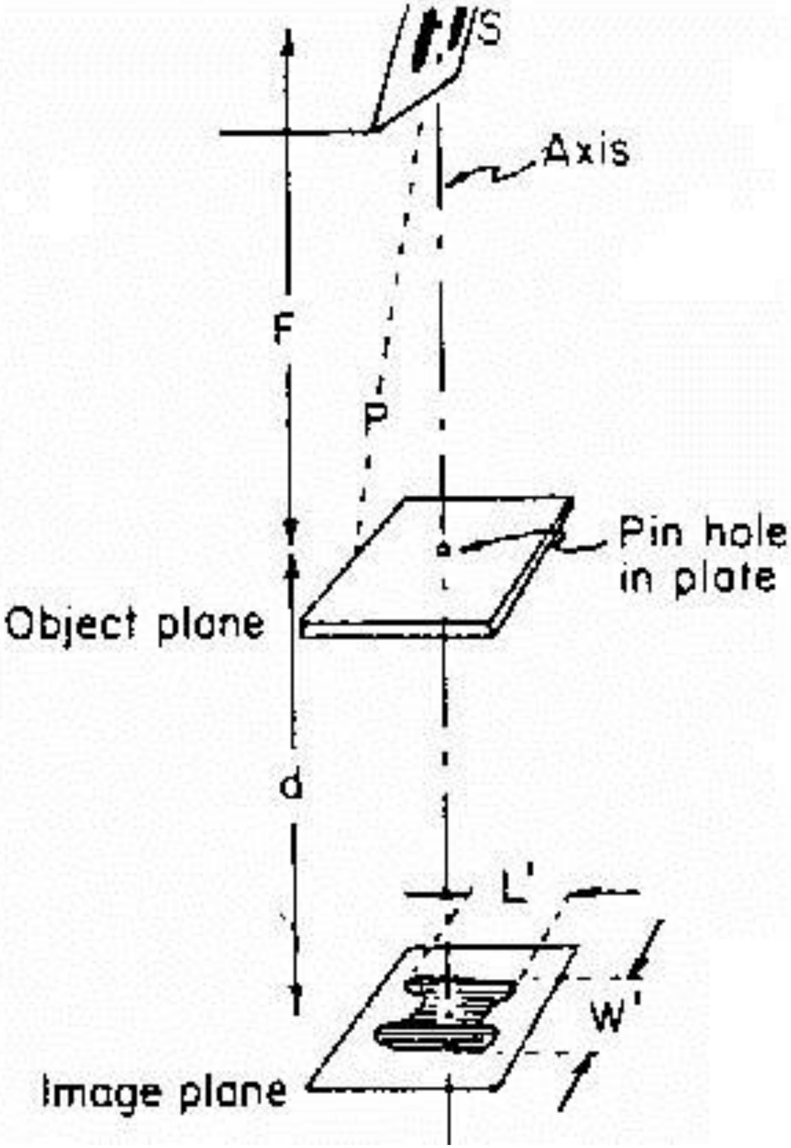
MTF of Film **MTF of Screen** **MTF of Target**



Star resolution pattern with and without Magnification

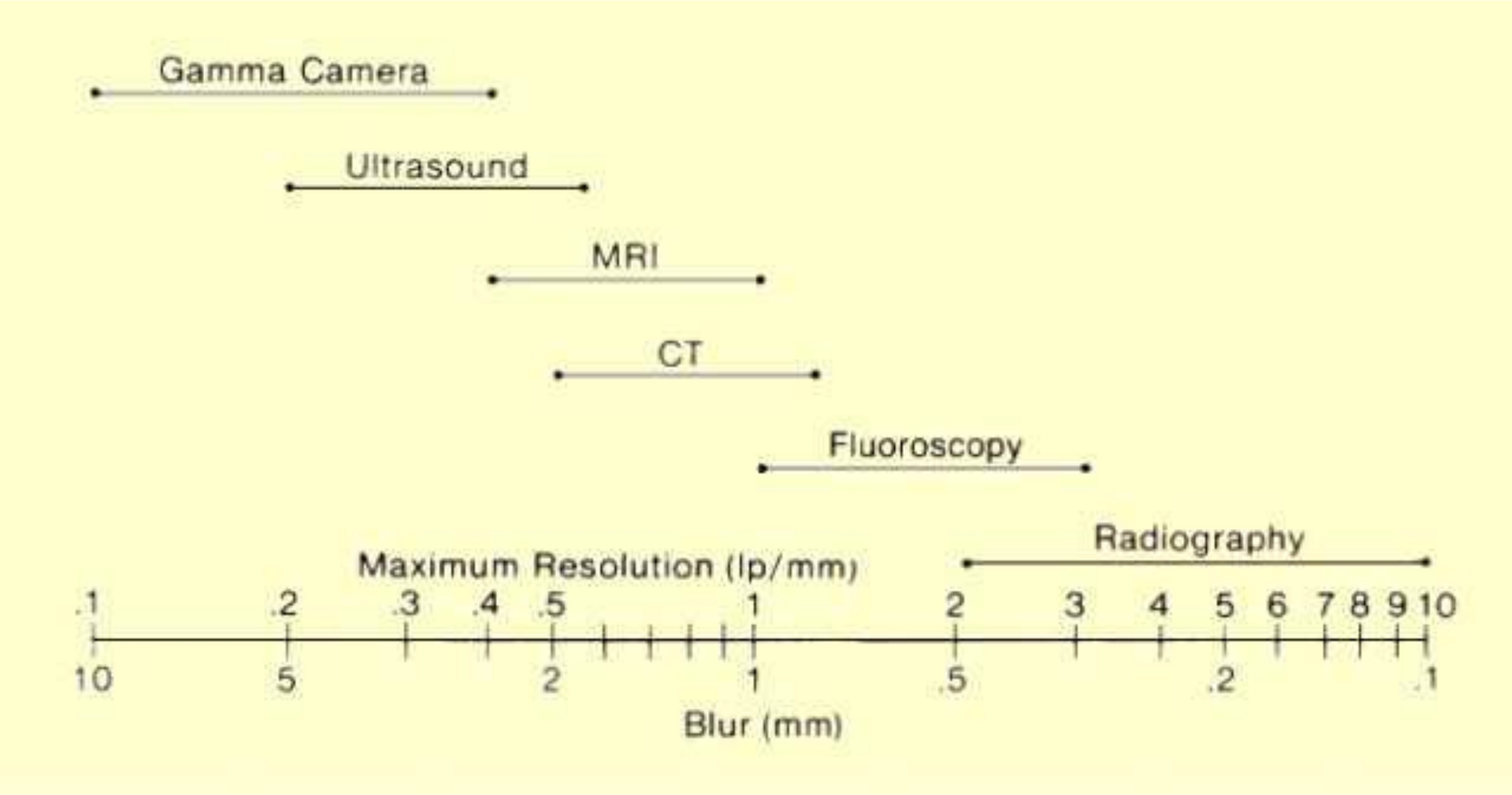


$$\theta = \frac{\pi f m a_0}{M}$$

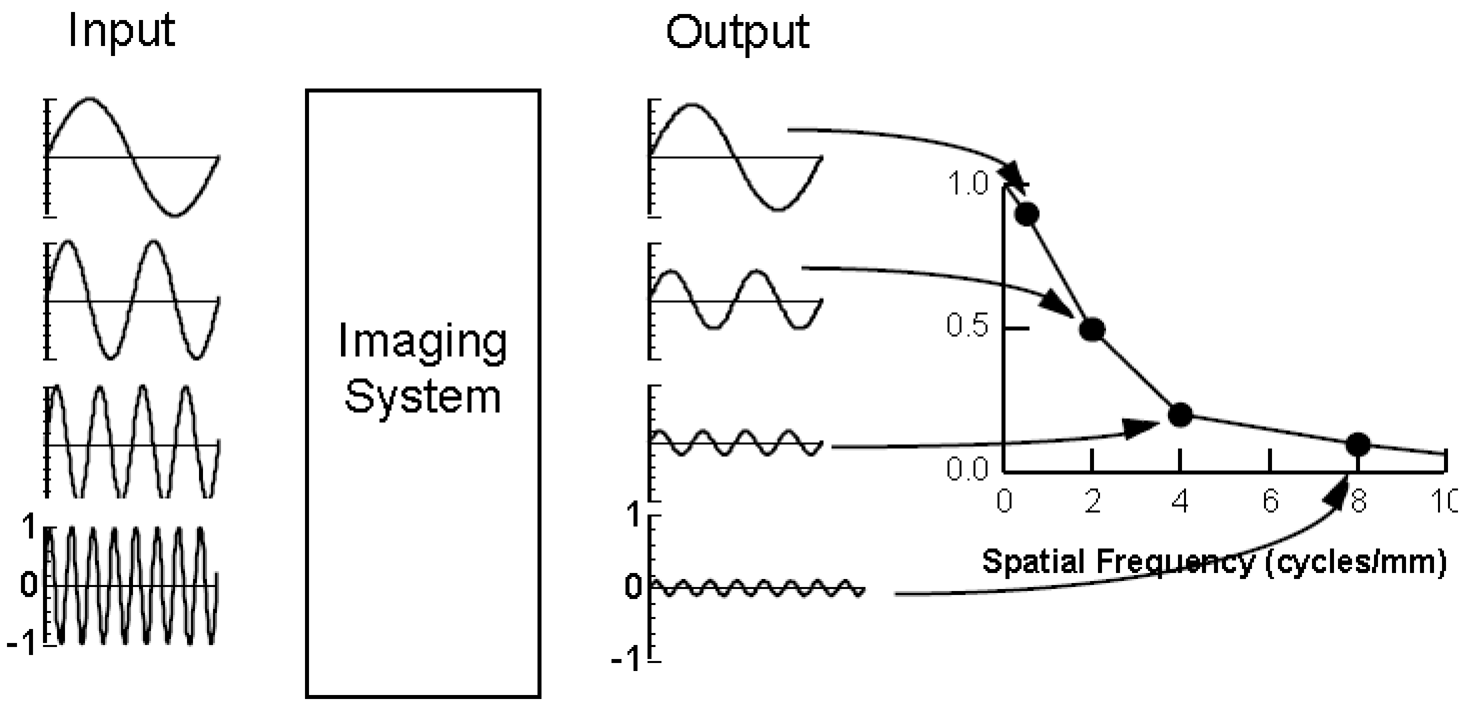


Change of focal spot size with tube loading

Expected Resolution of different modalities



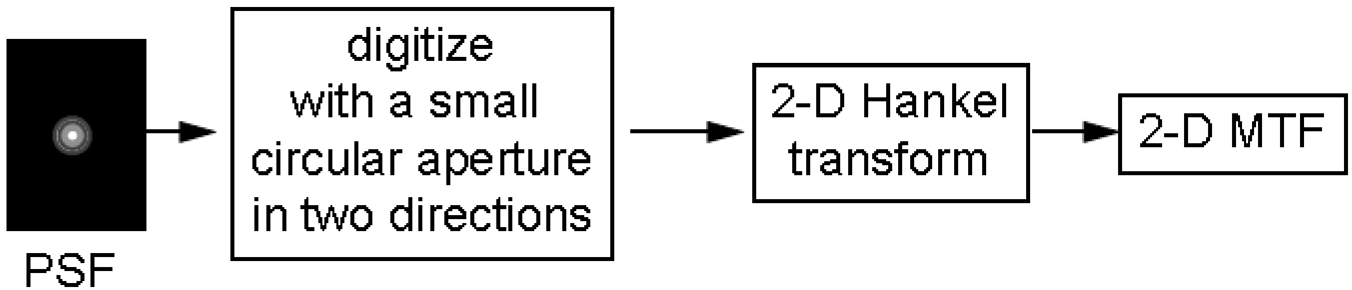
Measuring MTF (conceptually)



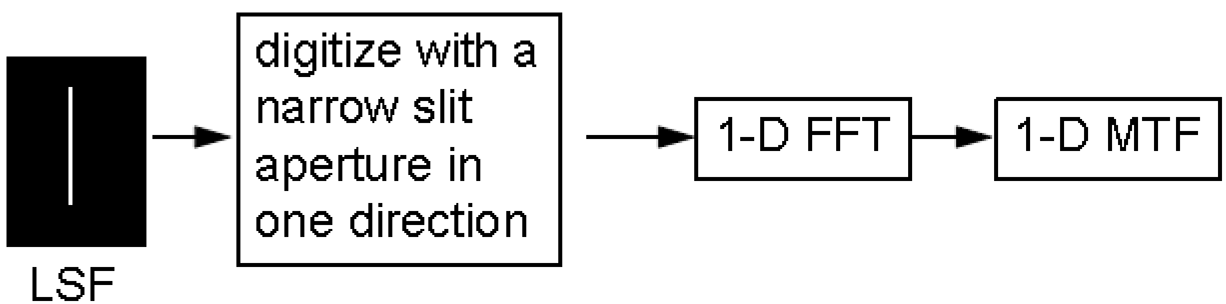
measures change in the <u>amplitude</u> of sine waves

Measuring MTF (theoretically)

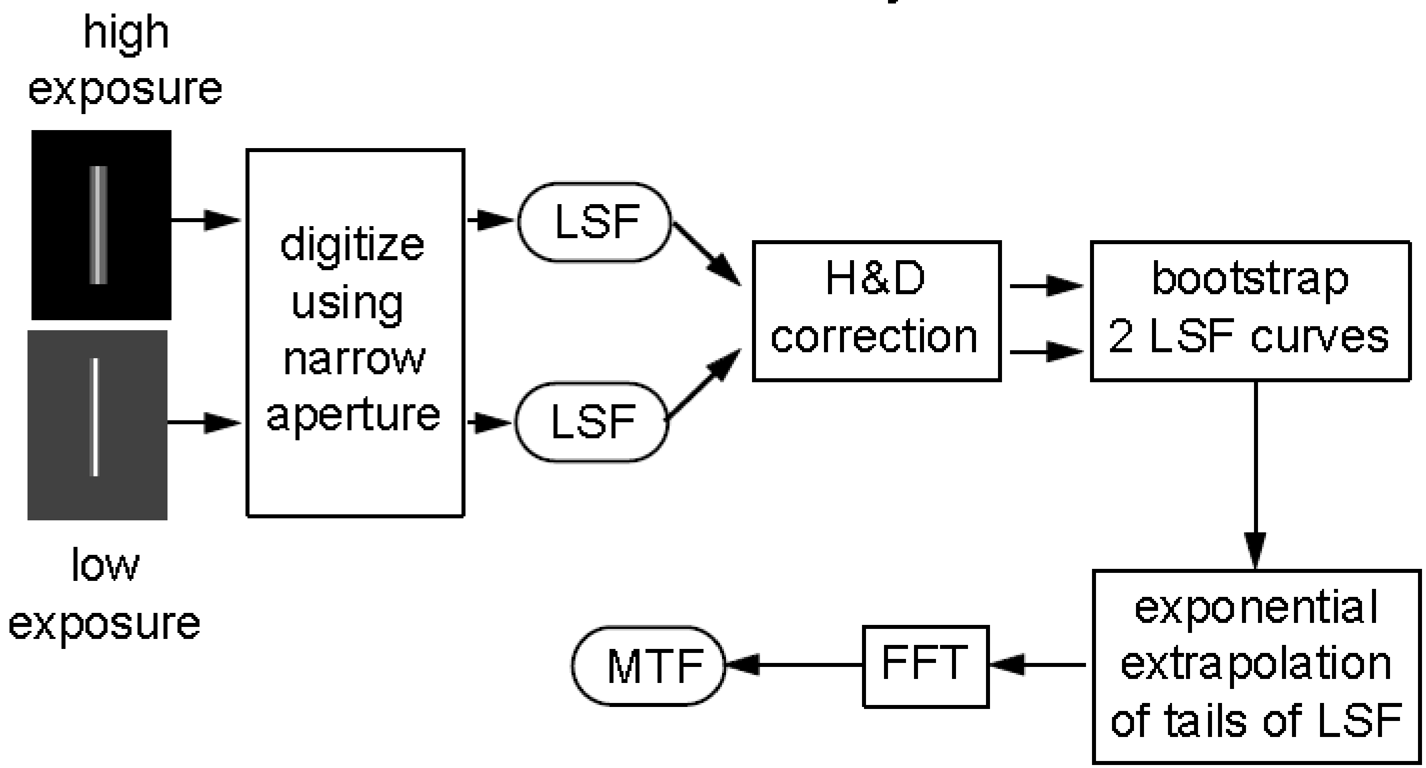
a POINT is composed of all spatial frequencies



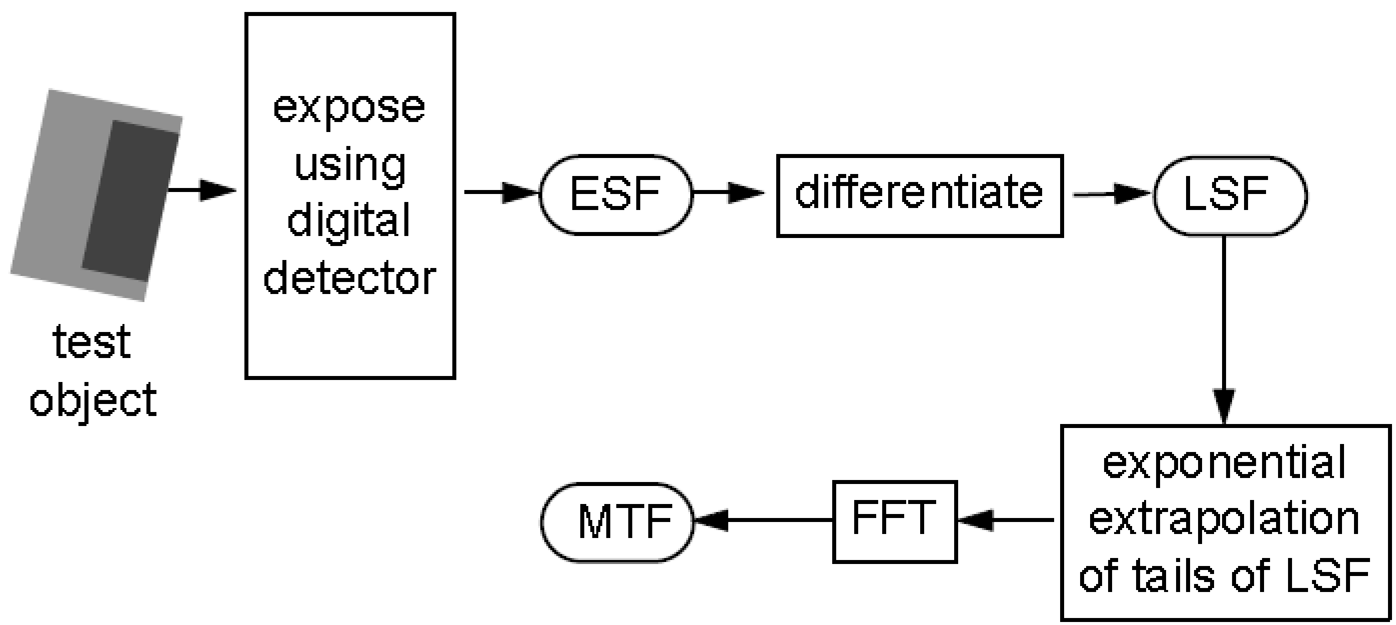
a LINE is composed of all spatial frequencies in one direction and zero frequency in the other

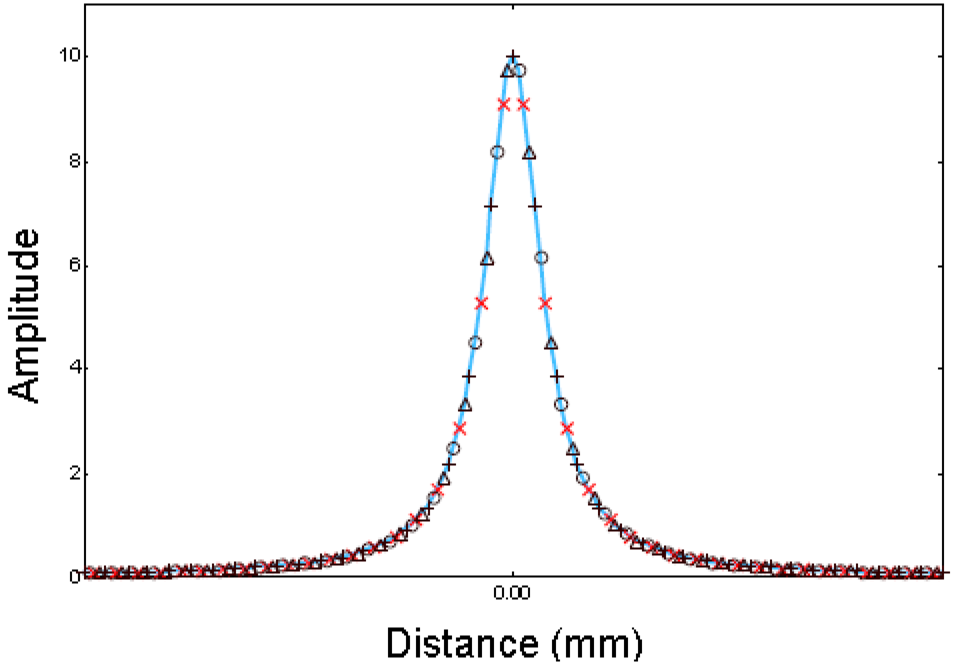


Measuring MTF (experimentally)
Screen-Film Systems

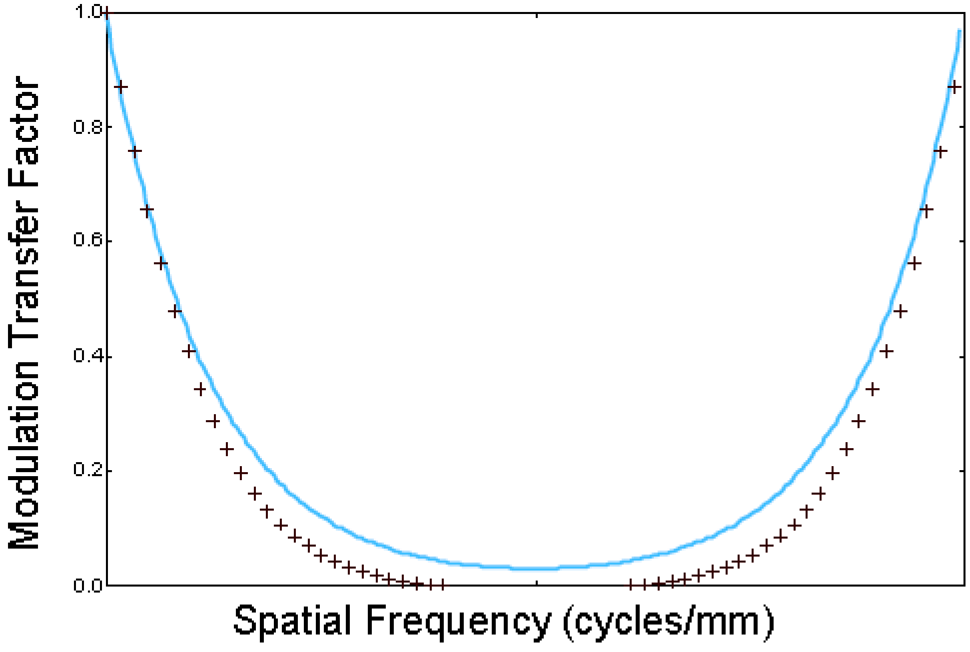


Measuring MTF (experimentally)
Digital Detectors (Pre-Sampled)





OVERSAMPLINY OF LSF



ALIASING