بسمه‌تعالي

Digital Image Processing

Image Enhancement in the Spatial Domain

（Chapter 4）

Objective

The principal objective of enhancement is to process an images so that the result is more suitable than the original image for a *SPECIFIC* application

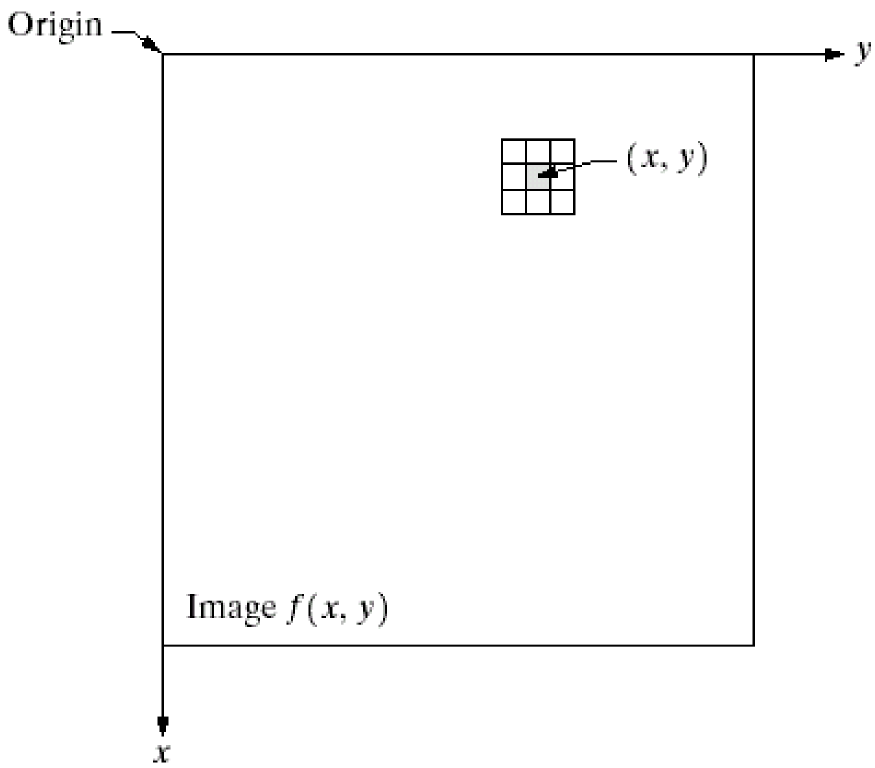
Category of image enhancement

- **Spatial domain**
- **Frequency domain**

Pixel neighborhood

$$g(x,y) = T\,[\,f(x,y)\,]$$

FIGURE 3.1 A 3×3 neighborhood about a point (x, y) in an image.



Point Processing, Gray-Level Transformation Function

$$s = T(r)$$

Contrast stretching

Thresholding

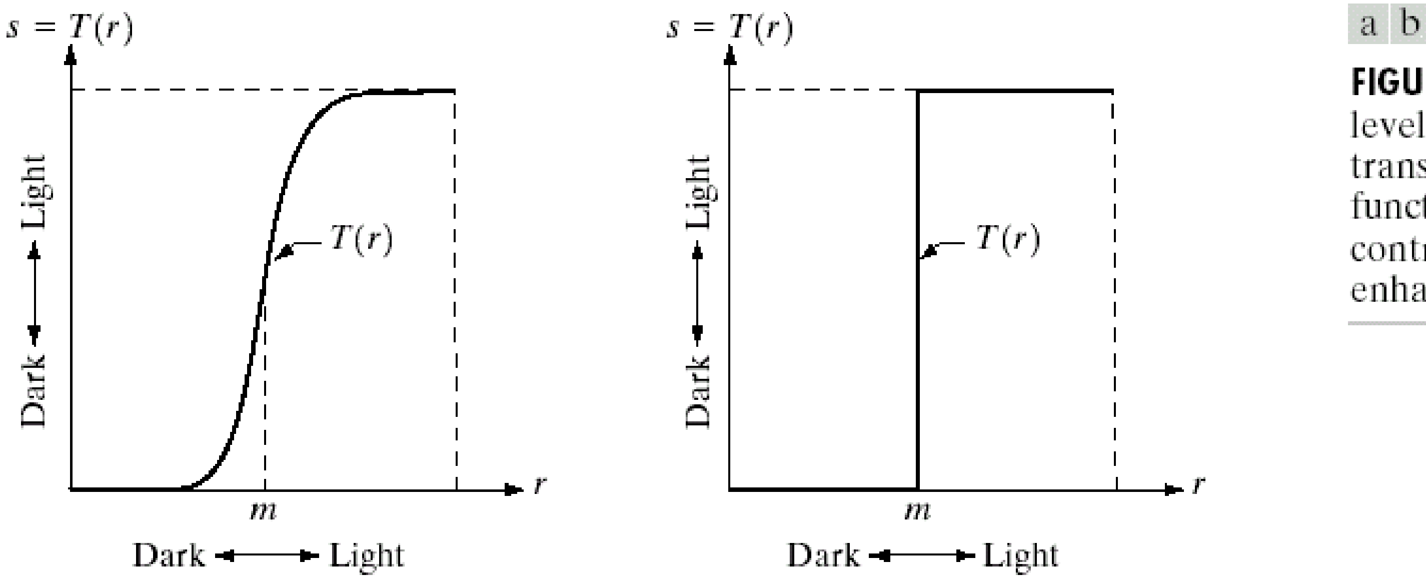


FIGURE 3.2 Gray-level transformation functions for contrast enhancement.

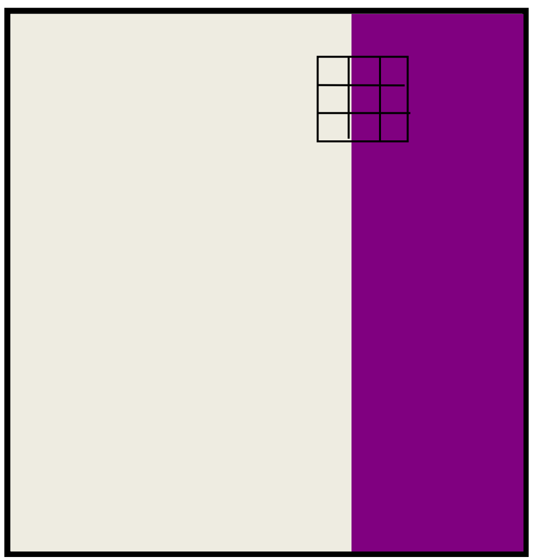
Mask Processing Filtering

$$g(x,y) = T\,[\,f(x,y)\,]$$

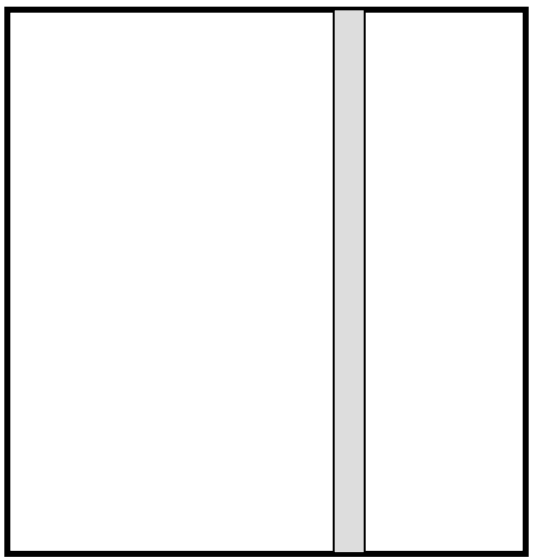
T

0	0	0
1	-1	0
0	0	0

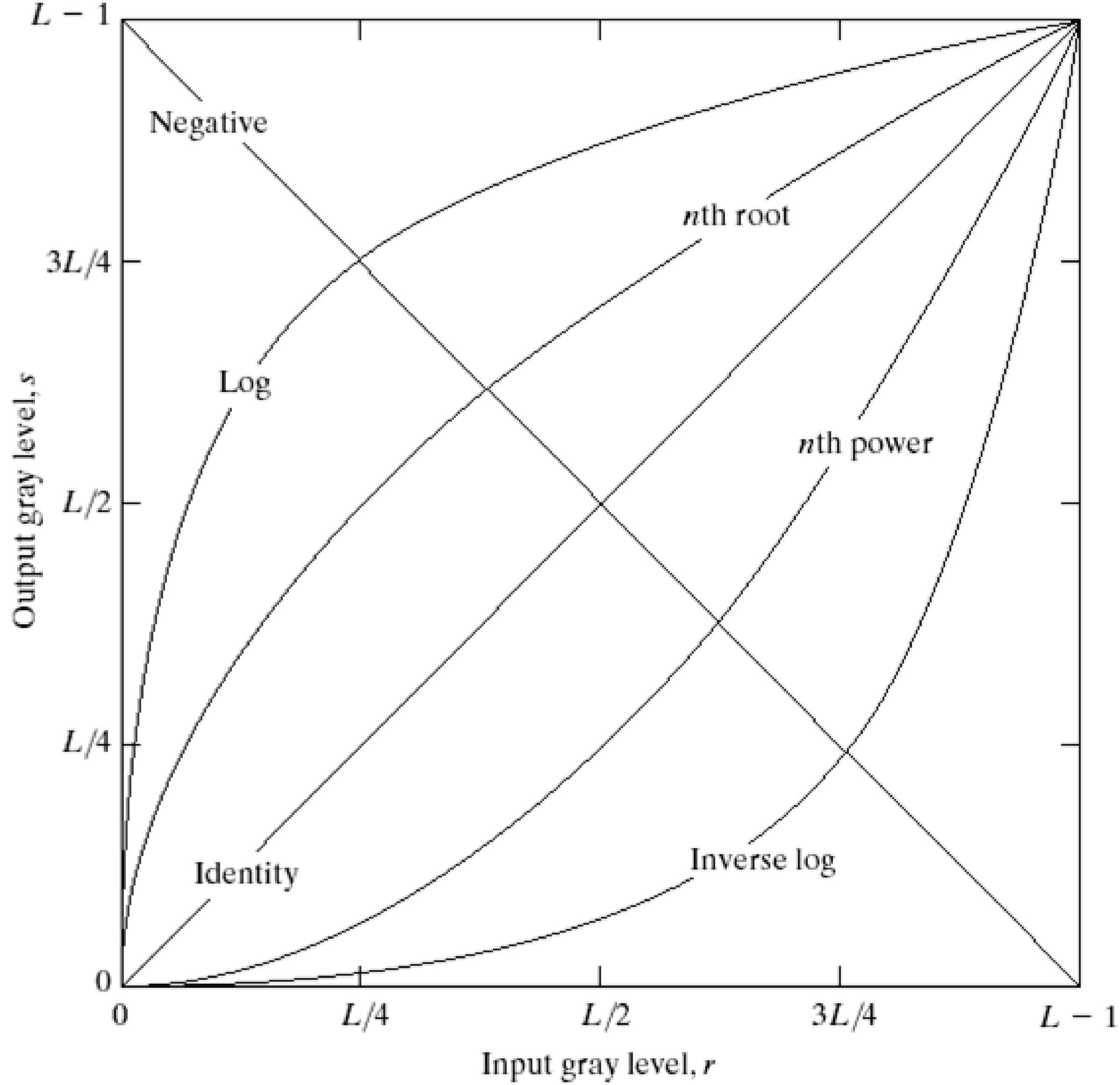
$f(x,y)$



$g(x,y)$



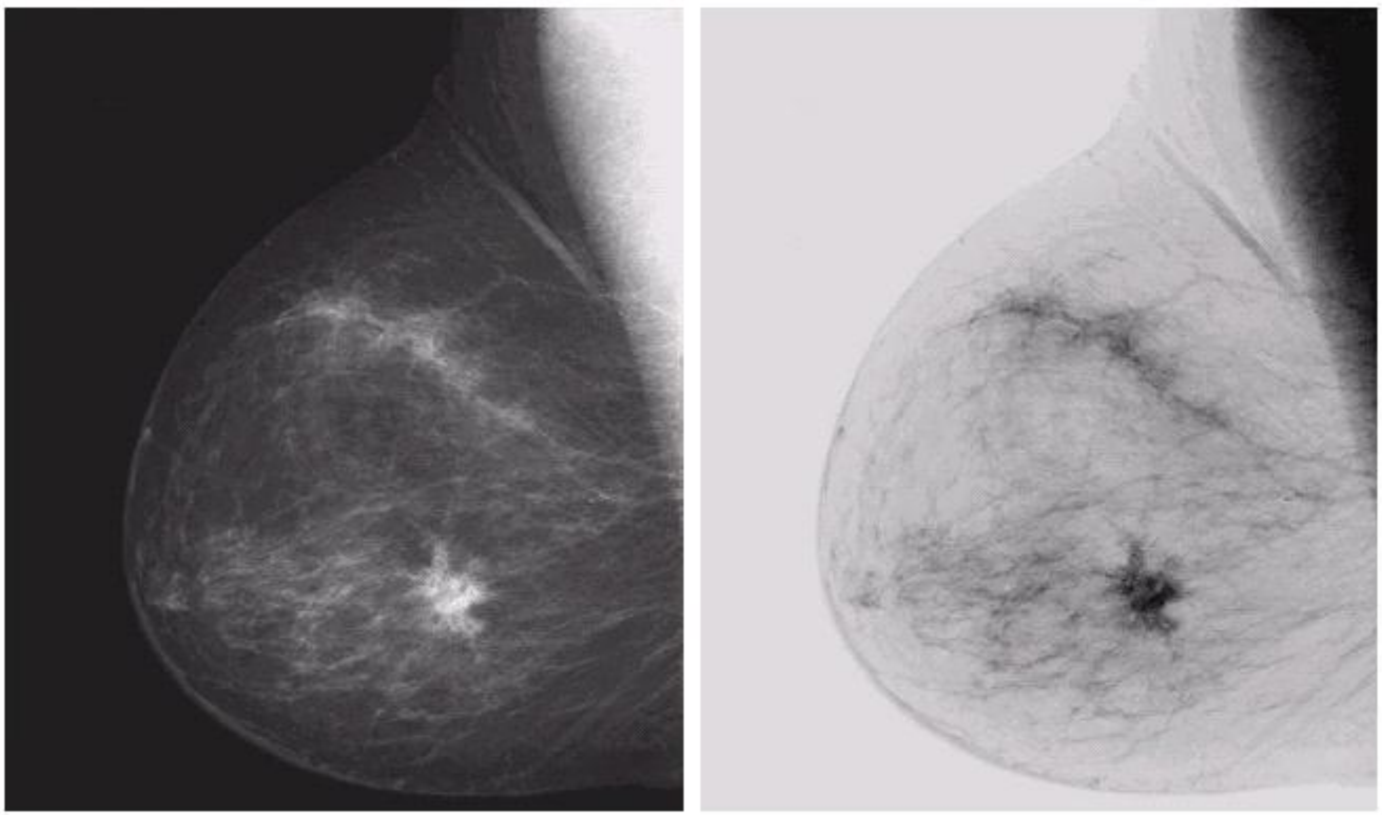
Basic Gray Level Transformation



Some Basic gray-Level transformation for image Enhancement

Image Negative

$$s = L - 1 - r$$

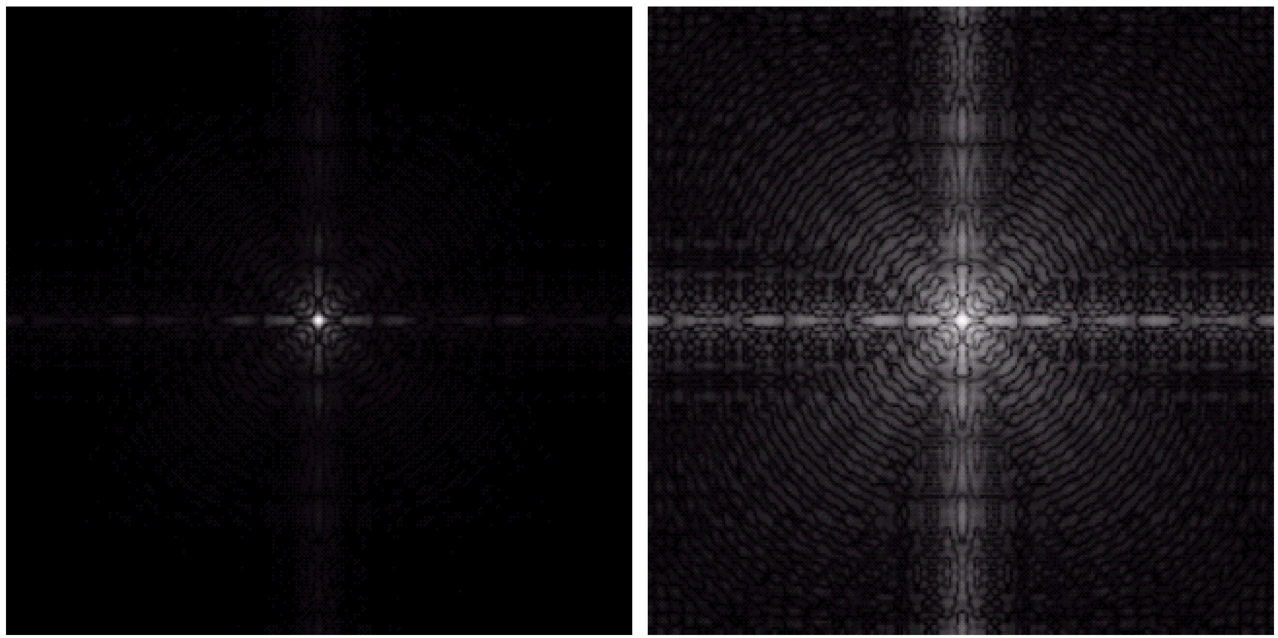


a b

FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

Log Transformations

$$s = c \log(1+r)$$



a b

FIGURE 3.5 (a) Fourier spectrum. (b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.

Pixel values dynamic range=[0 - 1.5×10^6]

Power-Law Transformations

$$s = cr^{\gamma}$$

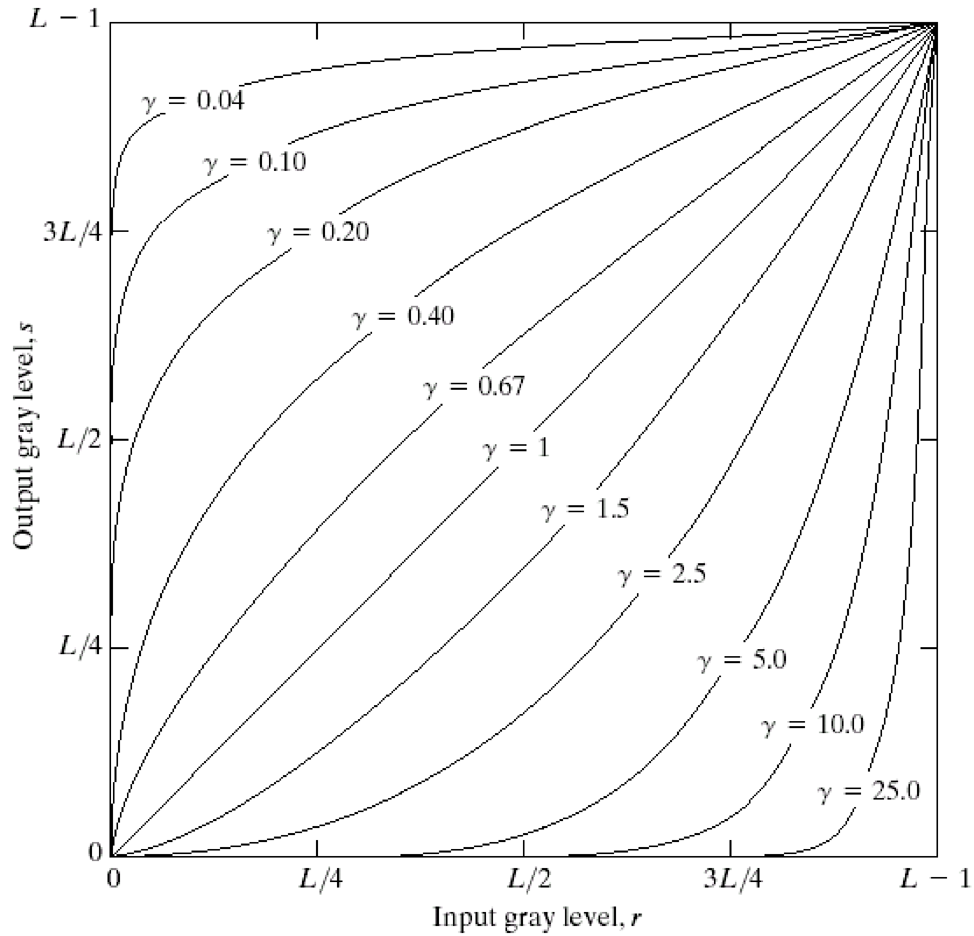


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases).

$$s = r^{2.5}$$

r	= [1	10	20	30	40	210	220	230	240	250	255]
$s(\gamma = 2.5)$	= [0	0	0	1	2	157	176	197	219	243	255]
$s(\gamma = .4)$	= [28	70	92	108	122	236	240	245	249	253	255]

Power-Law Transformations - Gamma Correction

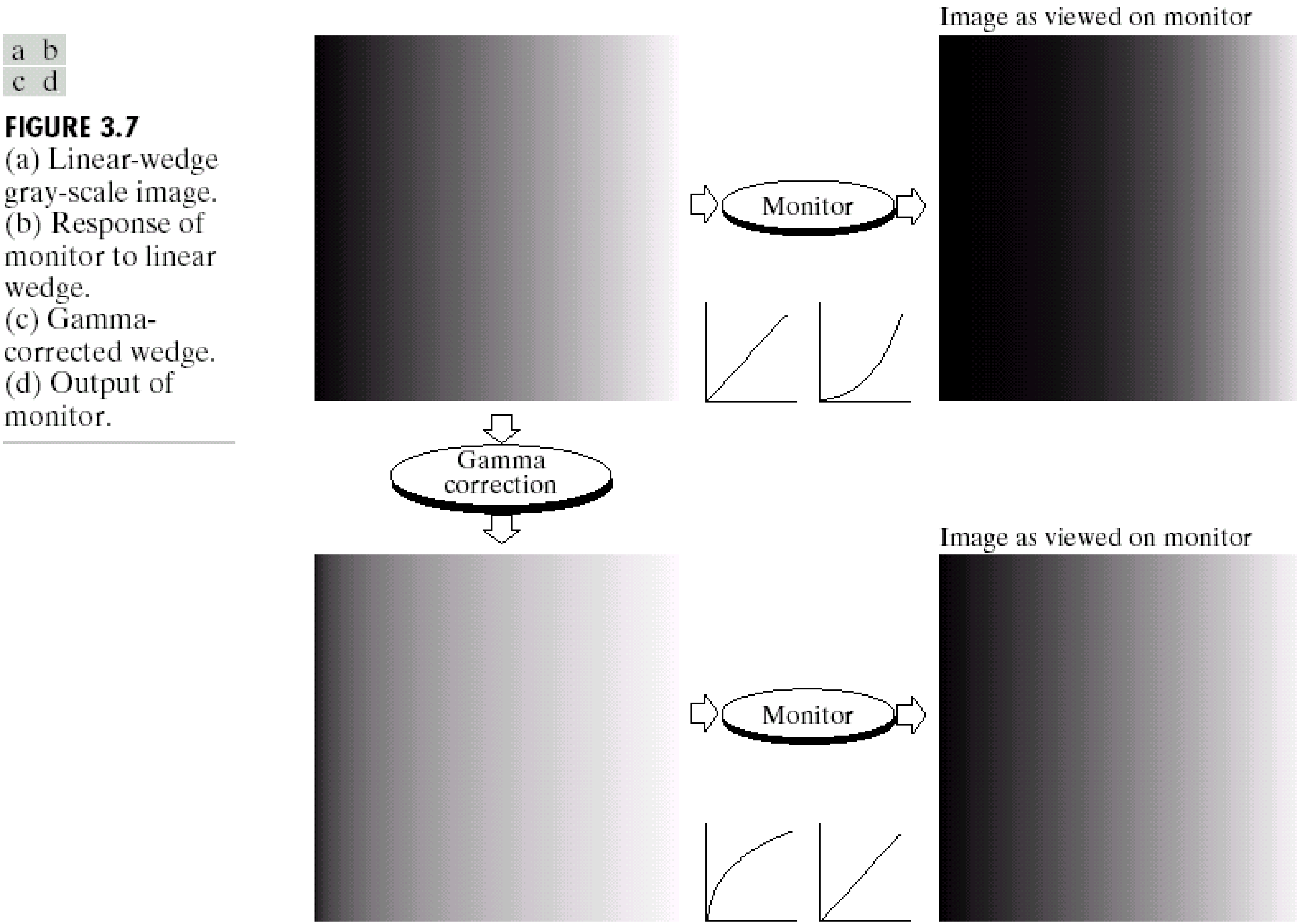


FIGURE 3.7 (a) Linear-wedge gray-scale image. (b) Response of monitor to linear wedge. (c) Gamma-corrected wedge. (d) Output of monitor.

Power-Law Transformations

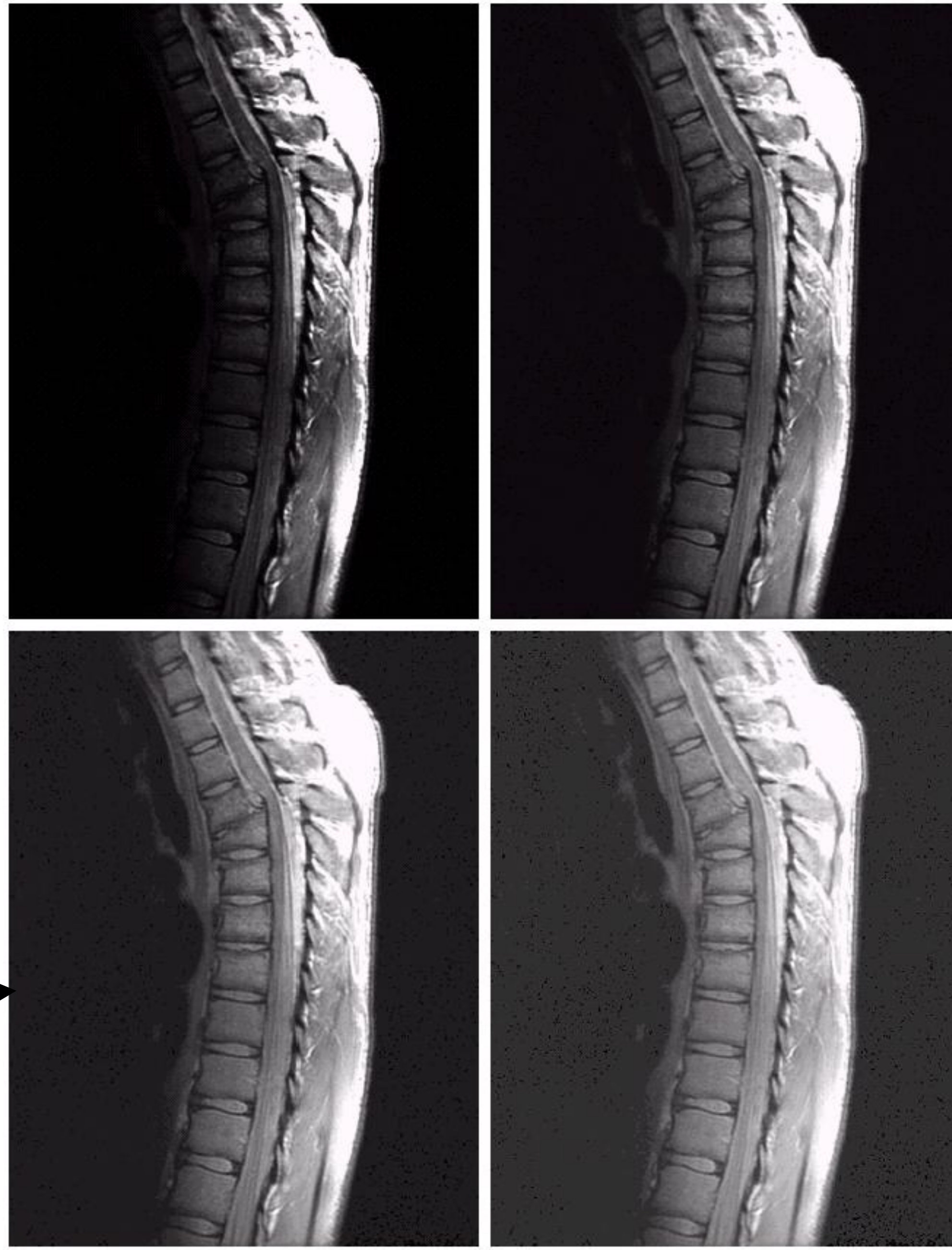


FIGURE 3.8 (a) Magnetic resonance (MR) image of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4$, and 0.3, respectively. (Original image for this example courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Power-Law Transformations

a b
c d

FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

$c=1$
$\gamma = 3$

$c=1$
$\gamma = 4$

$c=1$
$\gamma = 5$

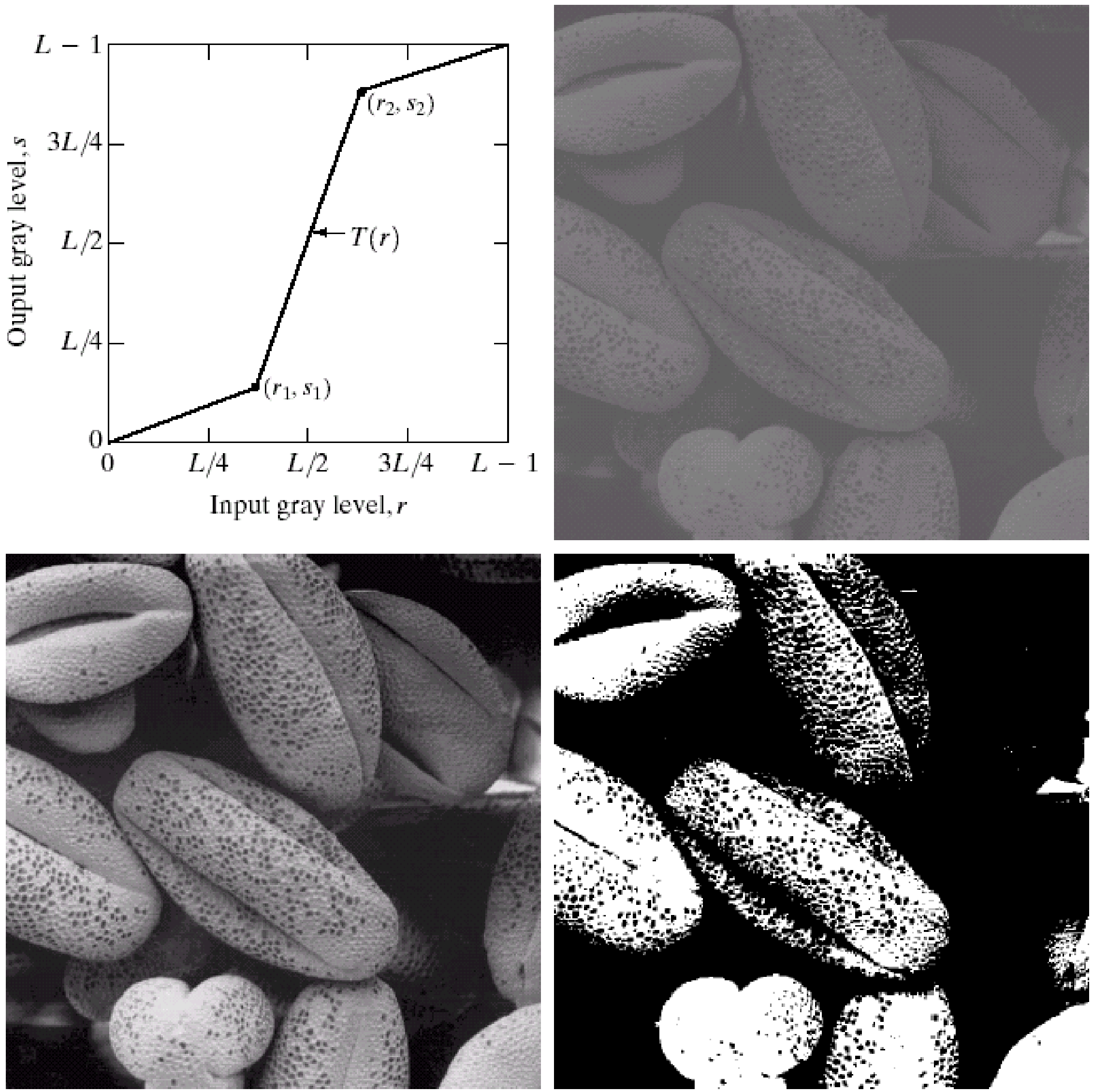
Piecewise-Linear Transformation Functions

- Advantage
  - Arbitrarily complex
- Disadvantage
  - More user input

- Type of Transformations
  - Contrast stretching
  - Gray-level slicing
  - Bit-plane slicing

Contrast Stretching

Objective: Increase the dynamic range of the gray levels



a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Causes for poor image

-Poor illumination

-Lack of dynamic range in the imaging sensor

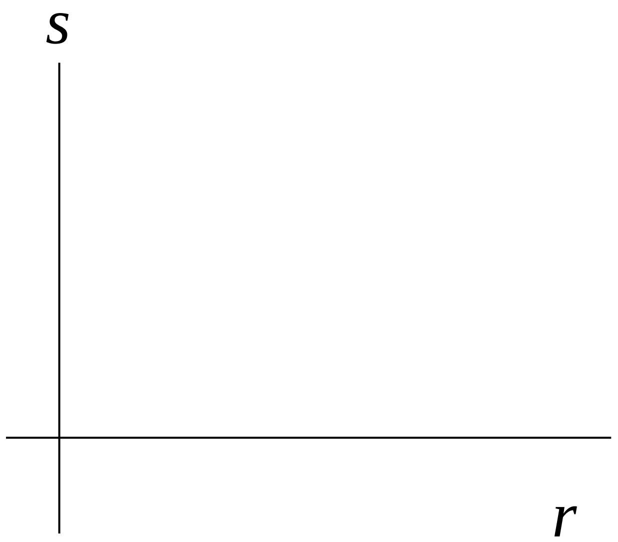
-Wrong lens aperture

Contrast Stretching

Example 1

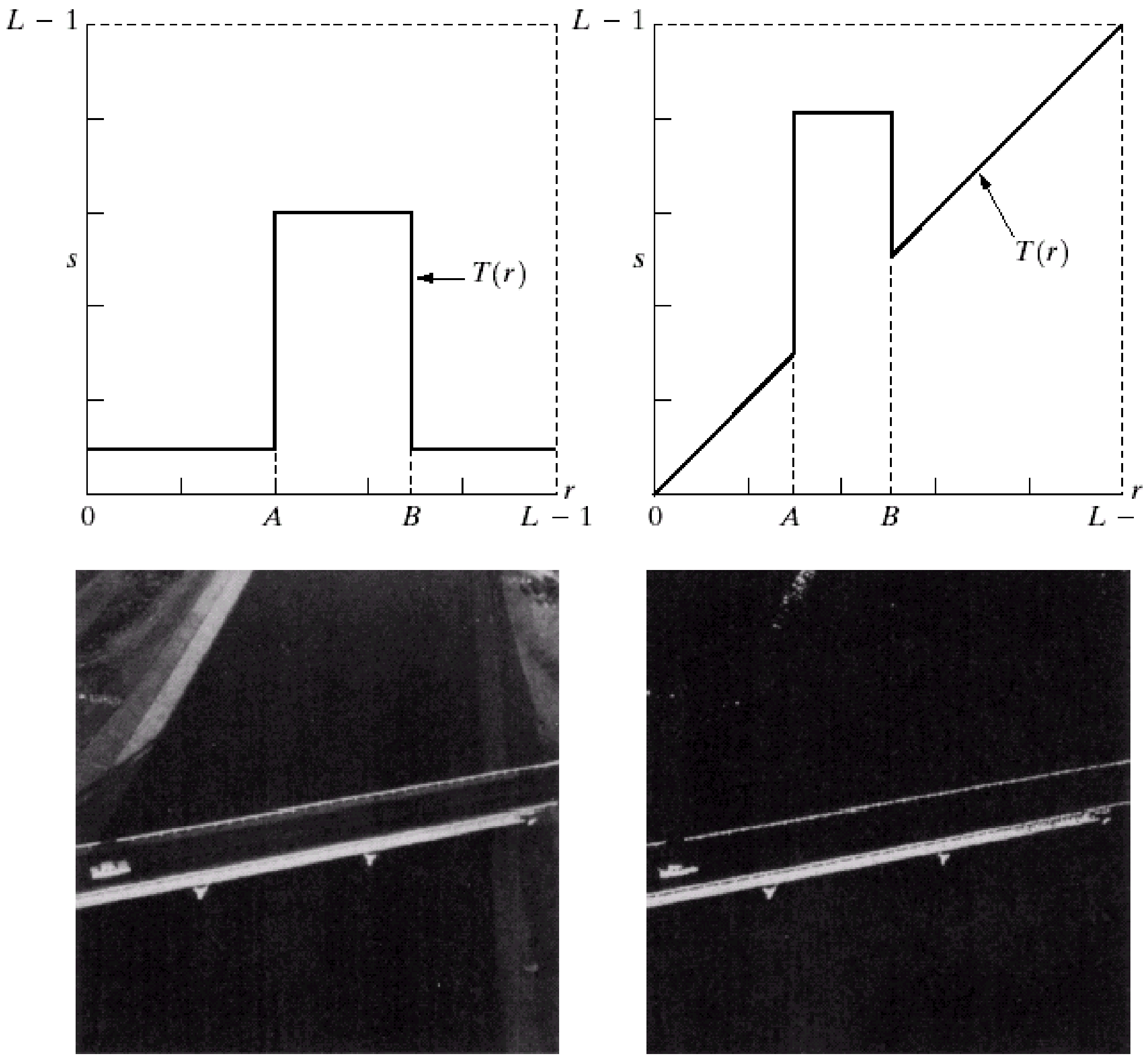
For image with intensity range [50 - 150]

What should (r_1, s_1) and (r_2, s_2) be to increase the dynamic range of the image to [0 - 255]?



Gray-Level Slicing

Objective: Highlighting a specific range of gray levels in an image.



a b
c d

FIGURE 3.11 (a) This transformation highlights range [A, B] of gray levels and reduces all others to a constant level. (b) This transformation highlights range [A, B] but preserves all other levels. (c) An image. (d) Result of using the transformation in (a).

Bit-Plane Slicing

255	138	30
65	12	201
180	111	85

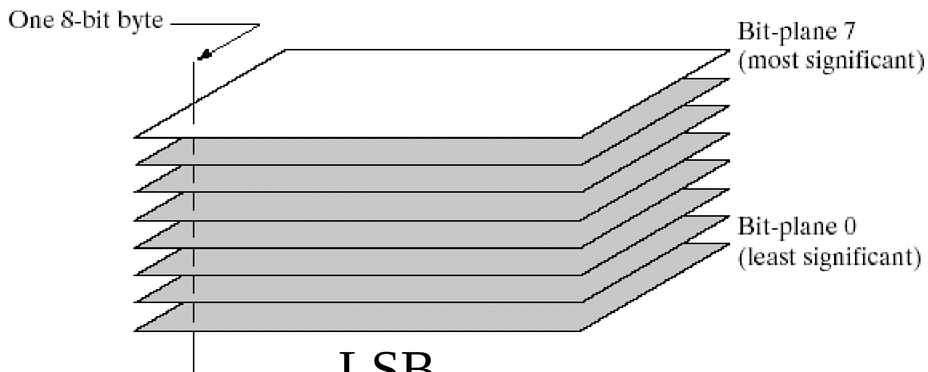


FIGURE 3.12 Bit-plane representation of an 8-bit image.

MSB plane

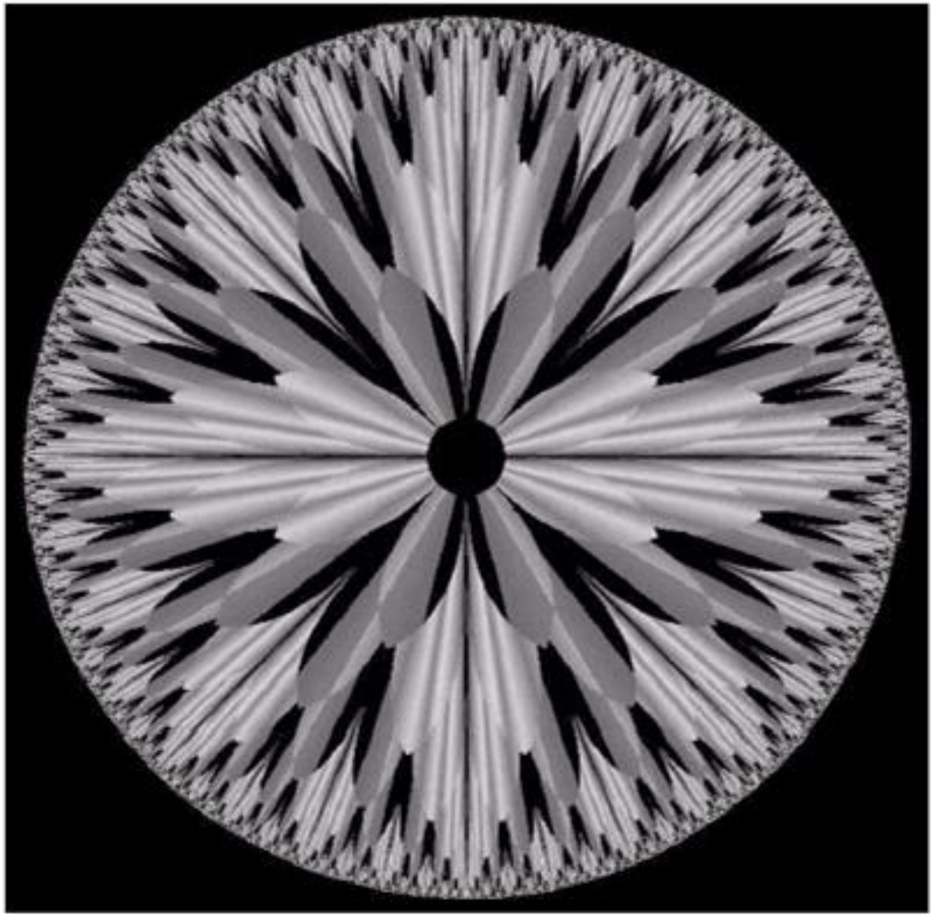
1	1	0
0	0	1
1	0	0

LSB plane

1	0	1
1	0	1
1	1	1

	LSB							MSB
255	1	1	1	1	1	1	1	1
138	0	1	0	1	0	0	0	1
30	1	1	1	1	1	0	0	0
65	1	0	0	0	0	0	1	0
12	0	0	1	1	0	0	0	0
201	1	0	0	1	0	0	1	1
183	1	1	1	0	1	1	0	1
111	1	1	1	1	0	1	1	0
85	1	0	1	0	1	0	1	0

8-bit fractal image used for Bit-Plane Slicing



Bit-Plane Slicing

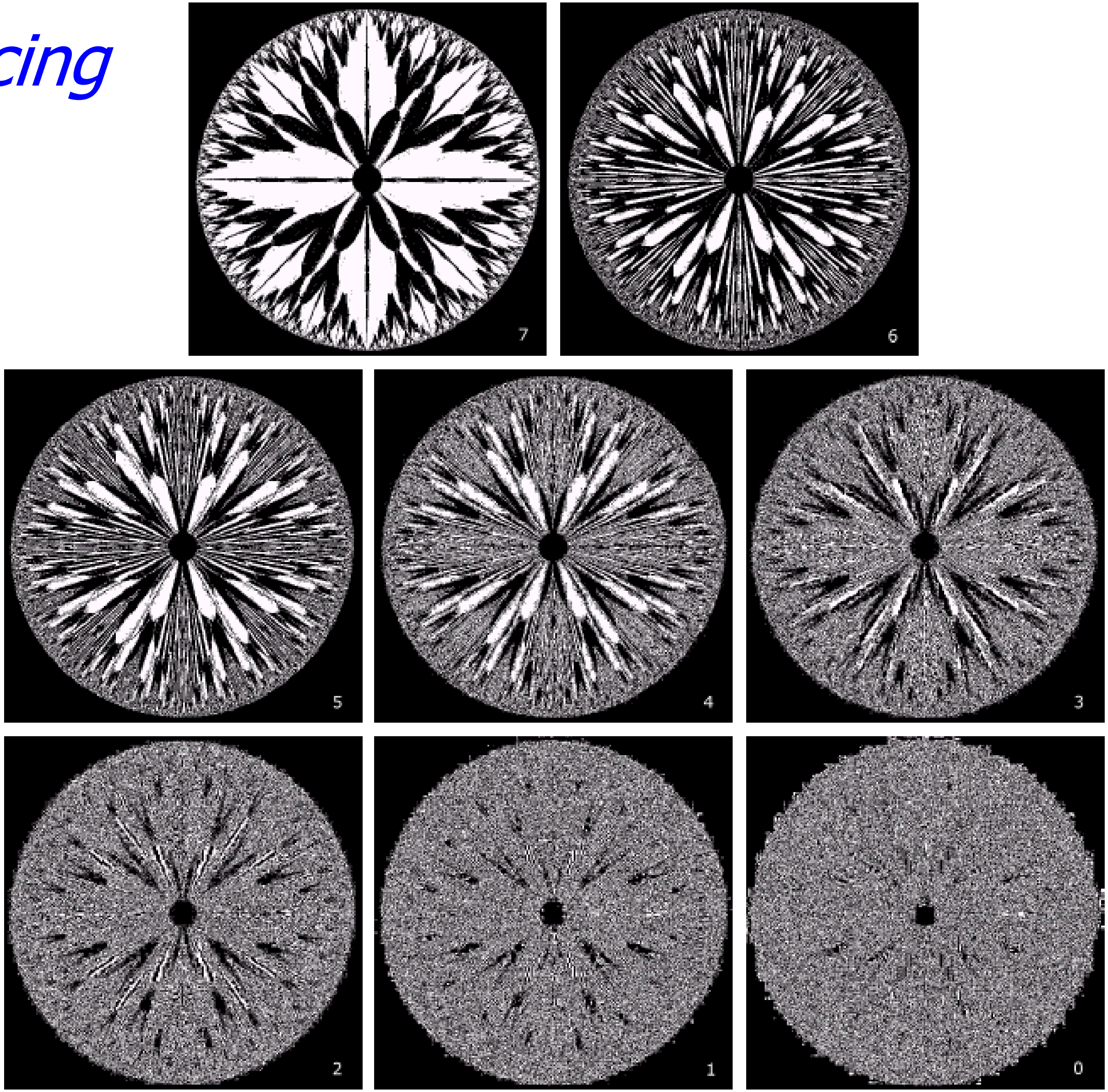


FIGURE 3.14 The eight bit planes of the image in Fig. 3.13. The number at the bottom, right of each image identifies the bit plane.

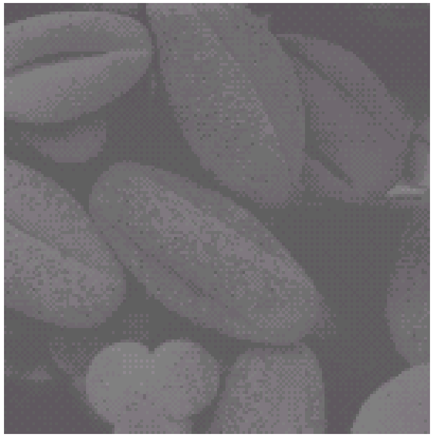
Histogram Processing

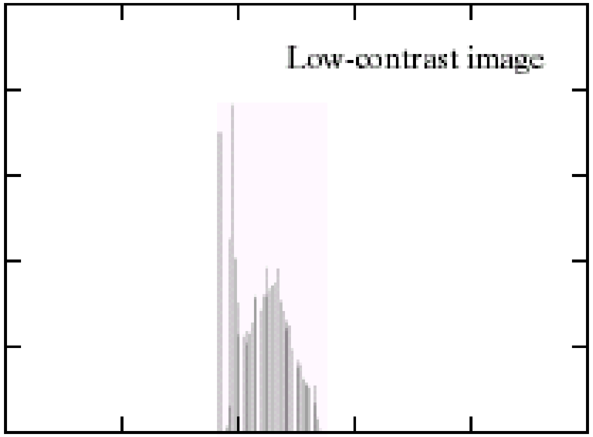
Histogram

The histogram of a digital image with gray levels in the range [0, L-1] is a discrete function

$$h(r_k) = n_k,$$

where r_k is the kth gray level and n_k is the number of pixels in the image having gray level r_k.





Normalized Histogram

Dividing each value of the histogram by the total number of pixels in the image, denoted by n.

$$p(r_k) = n_k / n.$$

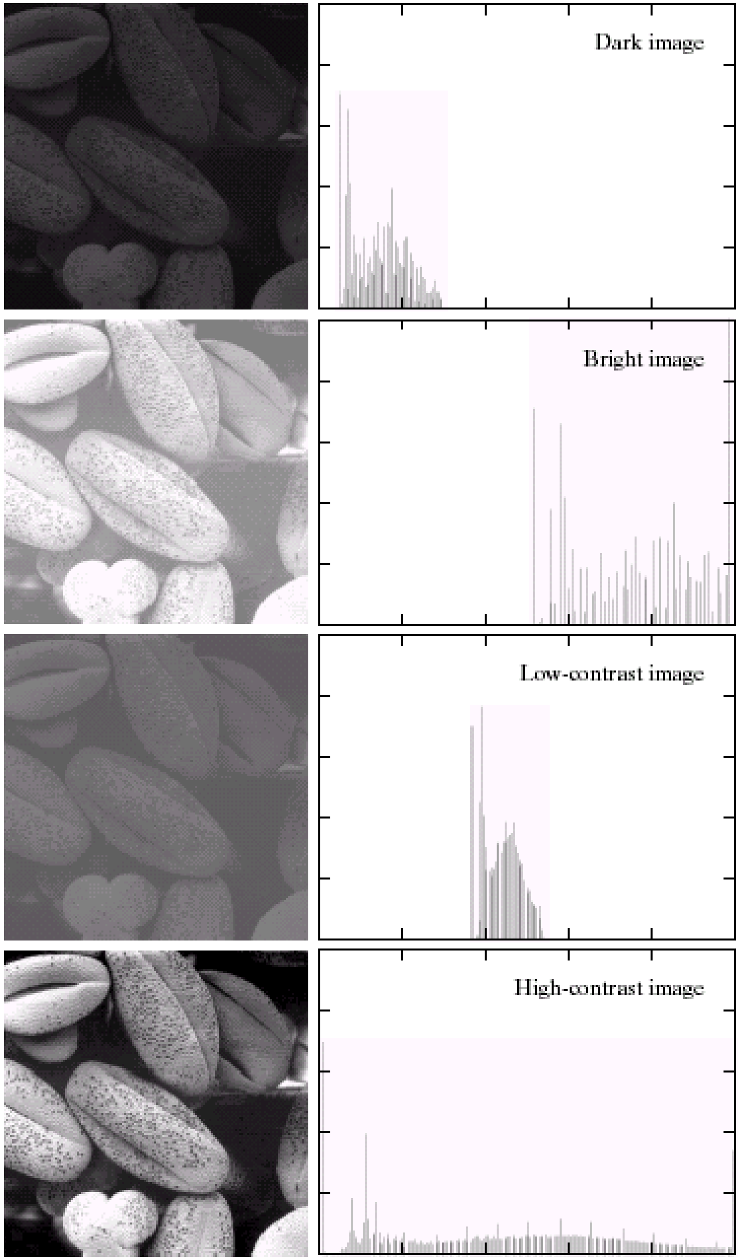
Normalized histogram provide useful image statistics.

Histogram Extraction using Matlab

```
function Normalized_Hist = Img_Hist(img)

[R,C]=size(img);
Hist=zeros(256,1);
for r = 1:R
    for c=1:C
        Hist(img(r,c)+1,1)=Hist(img(r,c)+1,1)+1;
    end
end
Normalized_Hist = Hist/(R*C);
plot(Normalized_Hist)
```

Histogram Processing



a b

FIGURE 3.15 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Histogram Equalization

Histogram equalization **is used to enhance image contrast and gray-level detail by spreading the histogram of the original image.**

$$s = T(r) \quad 0 \leq r \leq 1,$$

where r and s are normalized pixel intensities

Conditions for the transformation

(a) $T(r)$ is single-valued and monotonically increasing in the interval $0 \leq r \leq 1$

(b) $0 \leq T(r) \leq 1$ for $0 \leq r \leq 1$

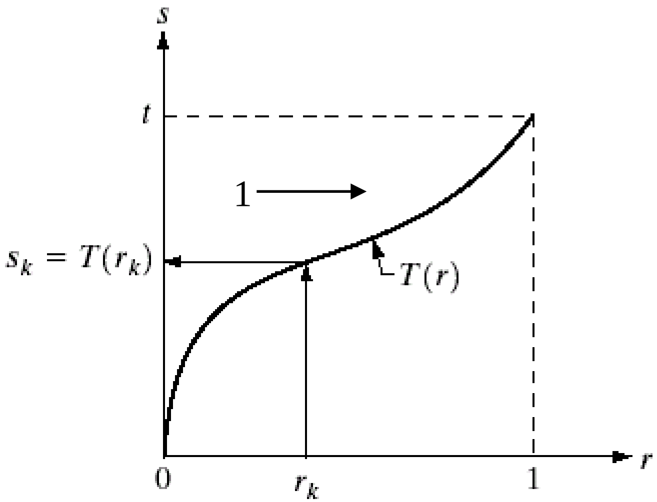


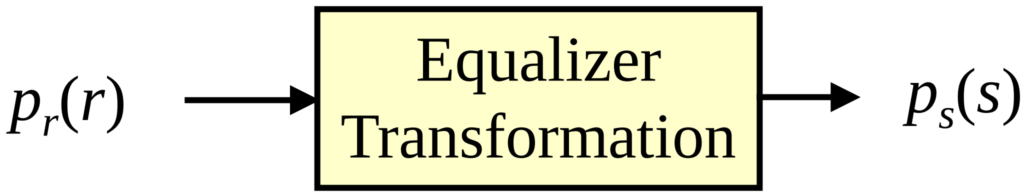
FIGURE 3.16 A gray-level transformation function that is both single valued and monotonically increasing.

Histogram Equalization

Objective of histogram equalization

Transform the histogram function of the original image $p_r(r)$ to a uniform histogram function.

$$p_s(s) = 1 \qquad 0 \leq s \leq 1$$



Continuous case

$$s = T(r) = \int_0^r p_r(w)dw$$

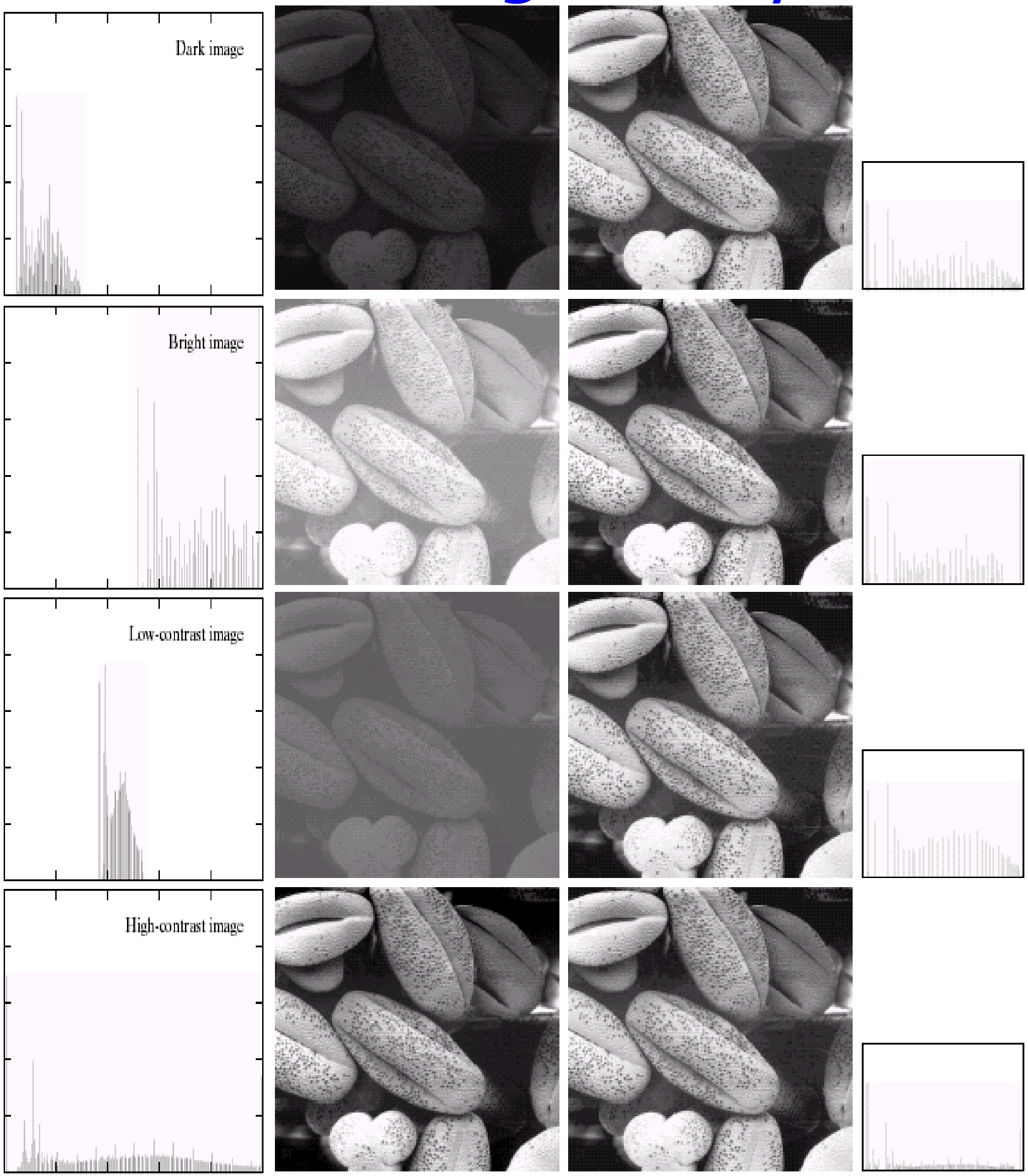
Discrete case

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j)$$

	Input image			Output image	
k		r_k	$p_r(r_k)$	s_k	
0		0	0	0	0
1		0.004	0	0	0
2		0.008	:	:	0
3		0.011	0	0	0
:		:	:	0	:
:		:	:	0	:
128		0.5	0.004	0.004	10
129	/255	0.505	0.15	0.154	39
130		0.51	0.05	0.204	52
:		:	:	:	:
:		:	:	:	:
253		0.992	0.005	0.989	252
254		0.996	0.006	0.994	253
255		1	0.004	1	255

$p_r(r_k) \xrightarrow{T(r_k)} s_k$, then s_k *255

Histogram Equalization

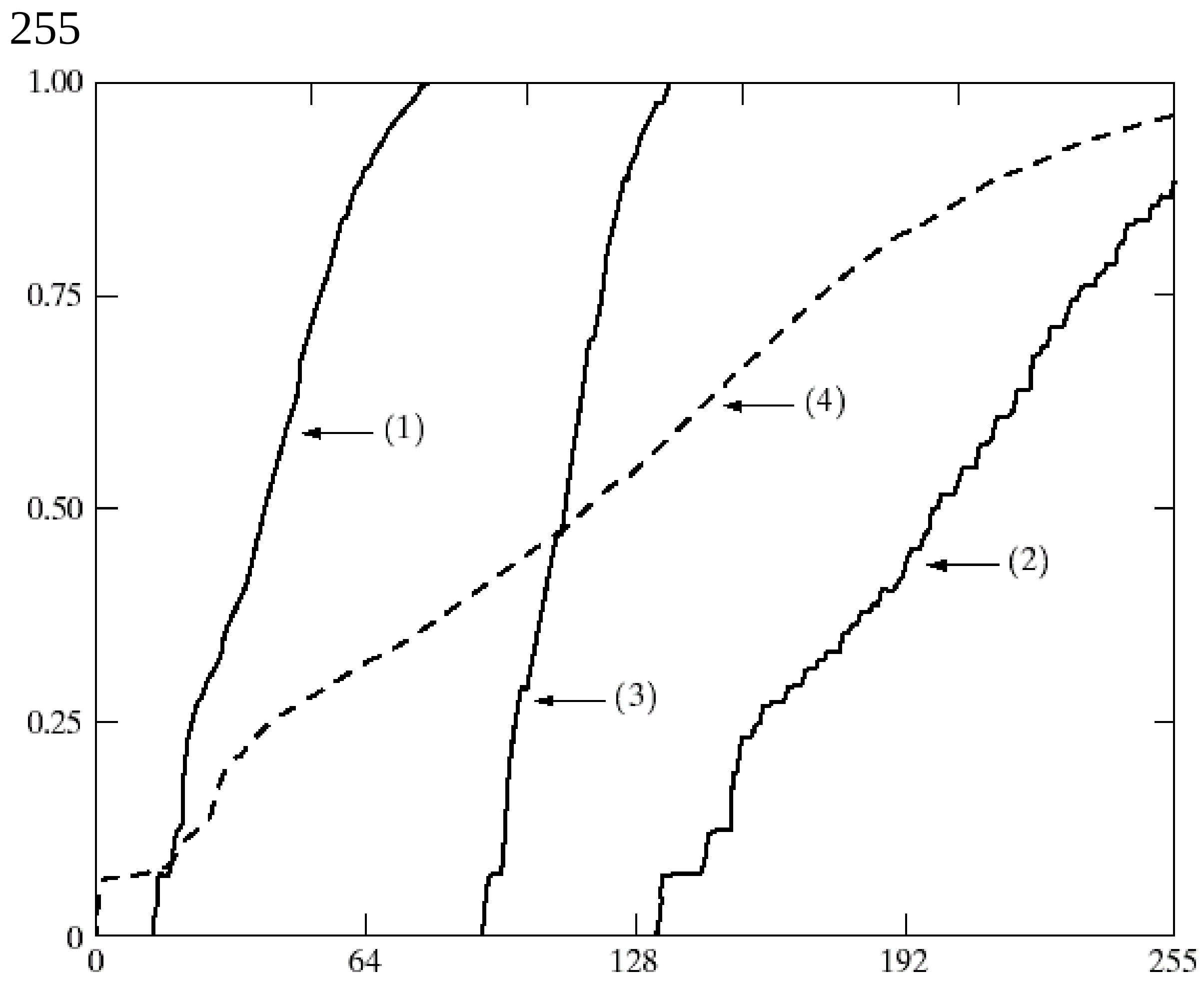


a b c

FIGURE 3.17 (a) Images from Fig. 3.15. (b) Results of histogram equalization. (c) Corresponding histograms.

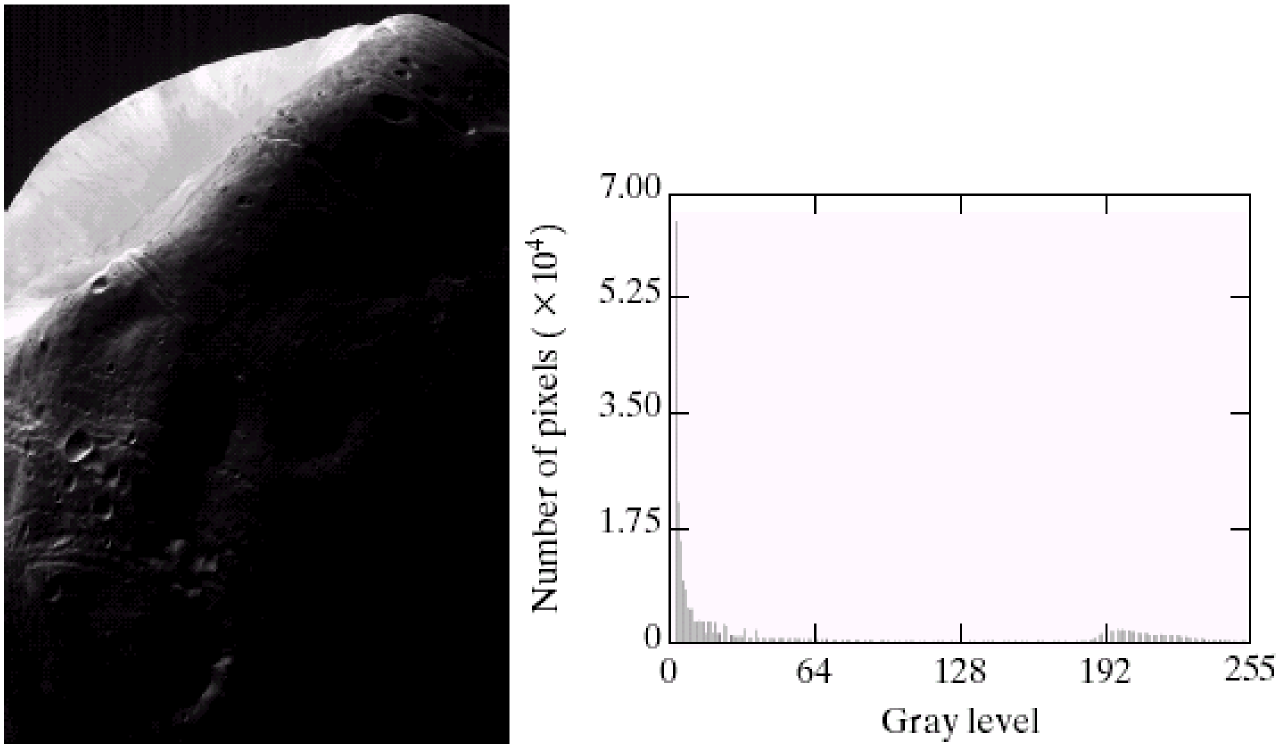
Histogram Equalization

FIGURE 3.18 Transformation functions (1) through (4) were obtained from the histograms of the images in Fig.3.17(a), using Eq. (3.3-8).



Histogram Matching

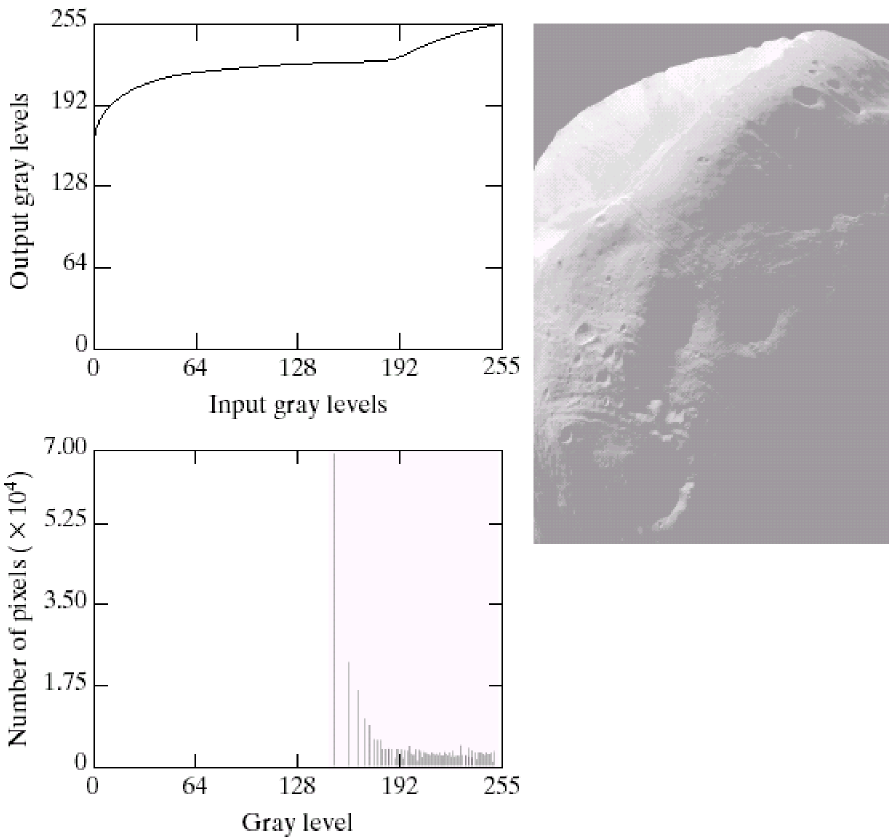
Is histogram equalization a good approach to enhance the image?



a b

FIGURE 3.20 (a) Image of the Mars moon Photos taken by NASA's *Mars Global Surveyor*. (b) Histogram. (Original image courtesy of NASA.)

Histogram Equalization



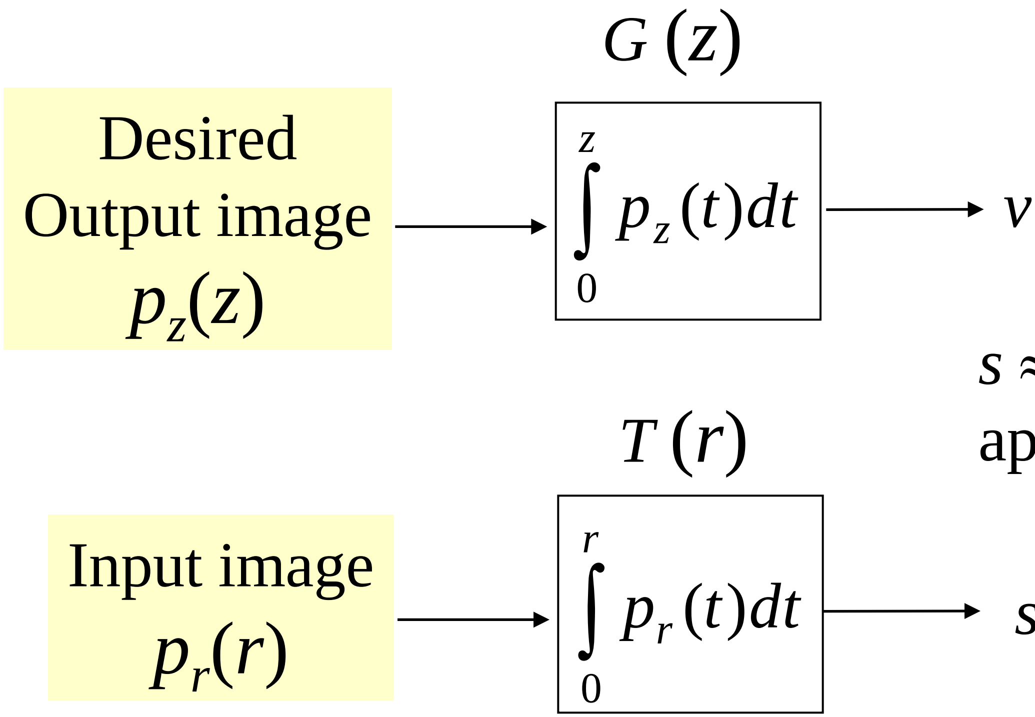
a b
c

FIGURE 3.21 (a) Transformation function for histogram equalization. (b) Histogram-equalized image (note the washed-out appearance). (c) Histogram of (b).

$p_r(r_k)$	s_k	s
0.6	0.6	153
0.12	0.72	184
0.08	0.8	204
0.0	0.8	204
:	:	:
0.01	0.9	230
0.005	0.95	242
0	1	255

Histogram Matching

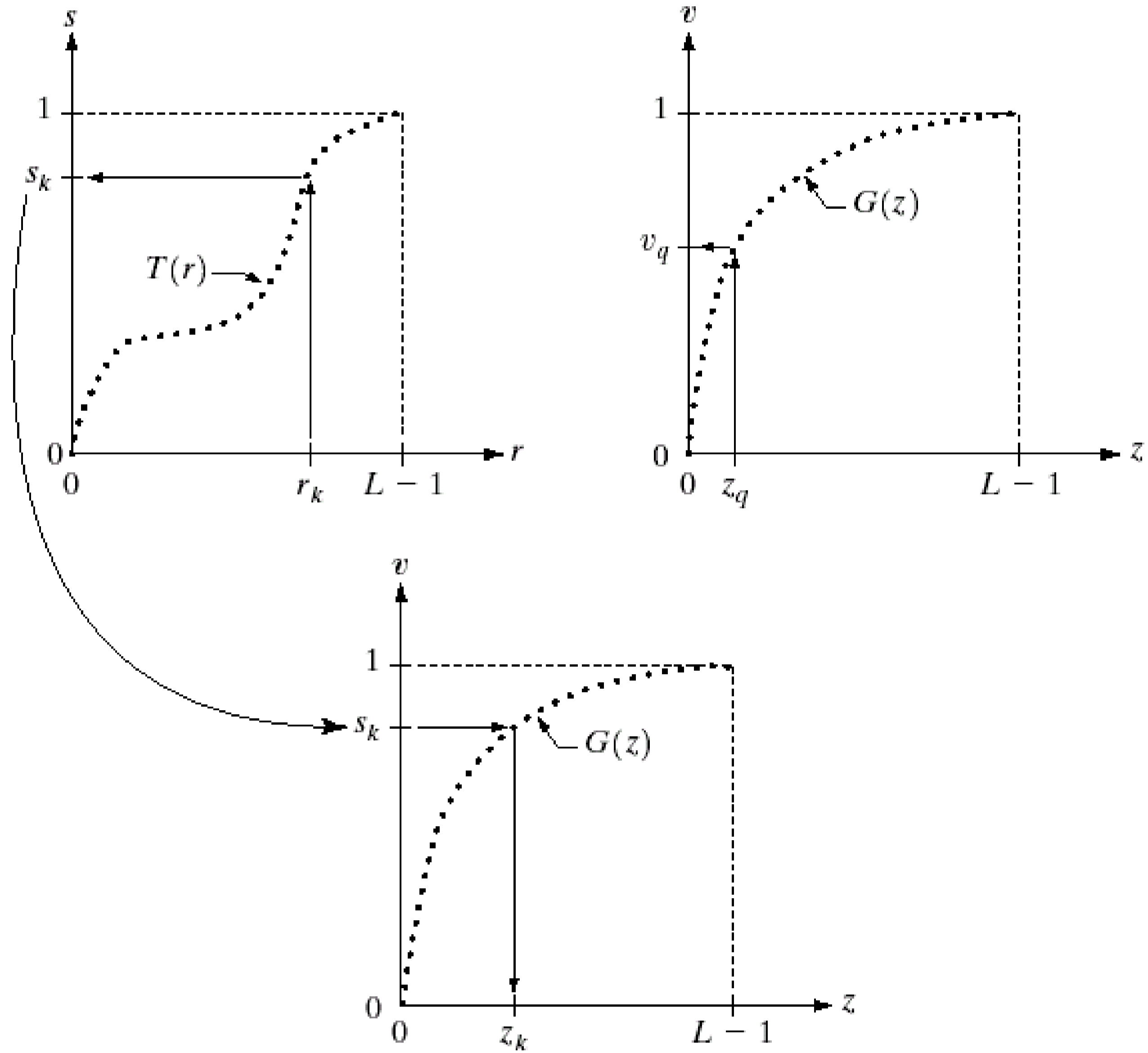
Histogram Matching method generates a processed image that has a specified histogram.



$s \approx v$, since both will have approximately uniform *pdf*

$z = G^{-1}[T(s)]$ $\quad$ $s = G[T(r)]$

Histogram Matching



a b
c

FIGURE 3.19 (a) Graphical interpretation of mapping from r_k to s_k via $T(r)$. (b) Mapping of z_q to its corresponding value v_q via $G(z)$. (c) Inverse mapping from s_k to its corresponding value of z_k.

Histogram Matching

1) Modify the histogram of the image to obtain $p_z(z)$.

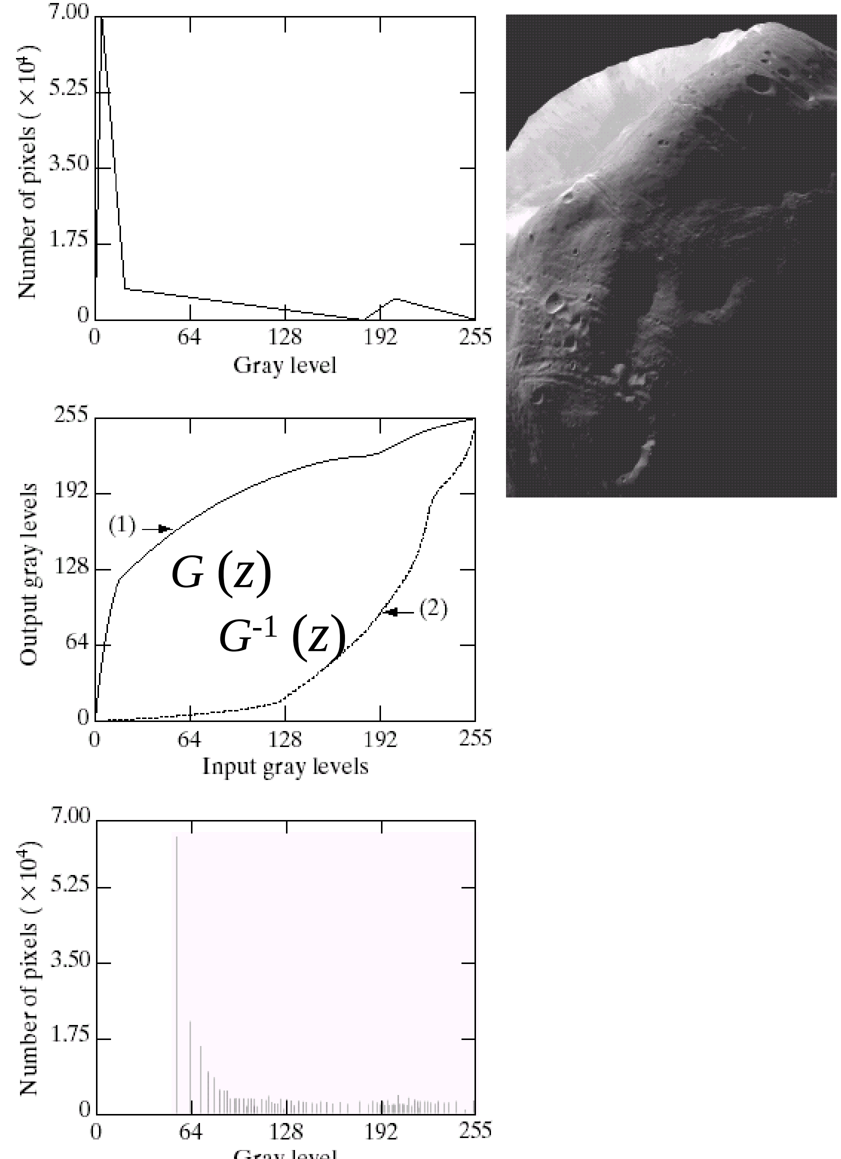
2) Find transformation function $G(z)$ using the modified histogram in step 1.

3) Find the inverse $G^{-1}(z)$

4) Apply G^{-1} to the pixels of the histogram-equalized image.

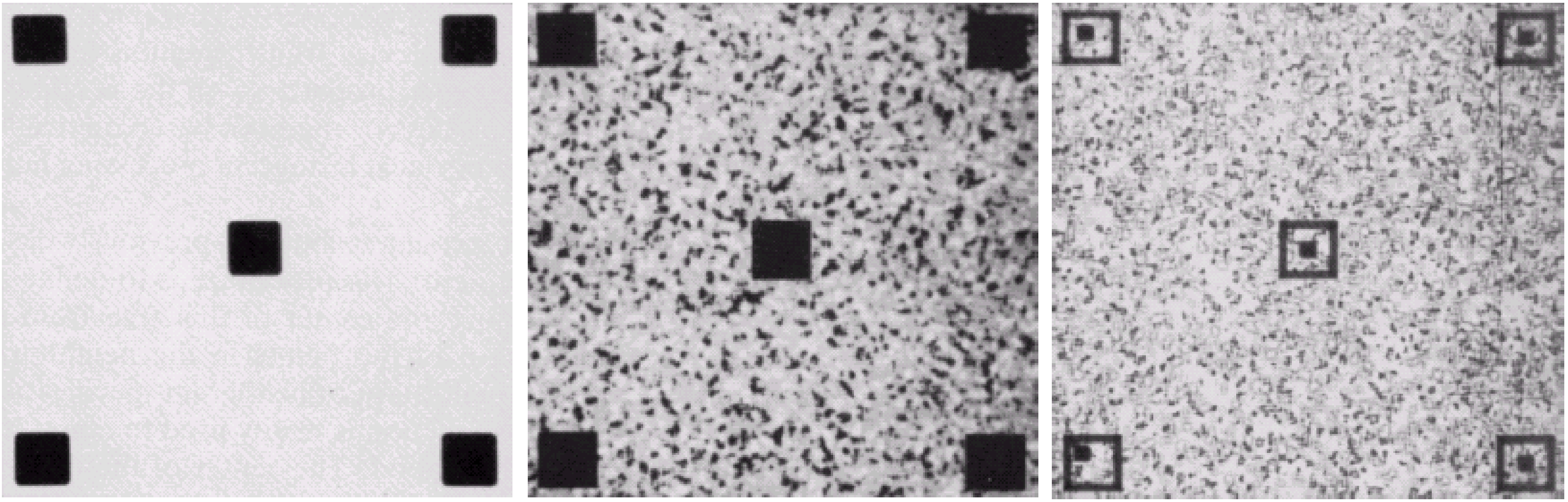
a	c
b	
d	

FIGURE 3.22 (a) Specified histogram. (b) Curve (1) is from Eq. (3.3-14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq. (3.3-17). (c) Enhanced image using mappings from curve (2). (d) Histogram of (c).



Local Histogram Enhancement

Local Histogram Enhancement



a b c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7 × 7 neighborhood about each pixel.

Histogram Statistics for Image Enhanc.

Contrast manipulation using local statistics, such as the mean and variance, is useful for images where part of the image is acceptable, but other parts may contain hidden features of interest.

FIGURE 3.24 SEM image of a tungsten filament and support, magnified approximately 130×. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).

Histogram Statistics for Image Enhanc.

Let (x,y) be the coordinates of a pixel in an image, and let S_{xy} denote a neighborhood (sub-image) of specified size, centered at (x,y).

$$m_{S_{xy}} = \sum_{(s,t)\in S_{xy}} r_{s,t}\, p(r_{s,t})$$

$$\sigma_{S_{xy}}^2 = \sum_{(s,t)\in S_{xy}} [r_{s,t} - m_{S_{xy}}]^2\, p(r_{s,t}).$$

The local mean and variance are the decision factors to whether apply local enhancement or not.

Histogram Statistics for Image Enhanc.

M_G: Global mean
D_G: Global standard deviation
E, k_0, k_1, k_2: Specified parameters

$$g(x,y)=\begin{cases} E.f(x,y) & \text{if } m_{S_{xy}} \le k_0 M_G \text{ AND } k_1 D_G \le \sigma_{S_{xy}} \le k_2 D_G \\ f(x,y) & \text{otherwise} \end{cases}$$

$E = 4$,
$k_0 = 0.4$,
$k_1 = 0.02$,
$k_2 = 0.4$
Size of local area = 3×3

Histogram Statistics for Image Enhanc.

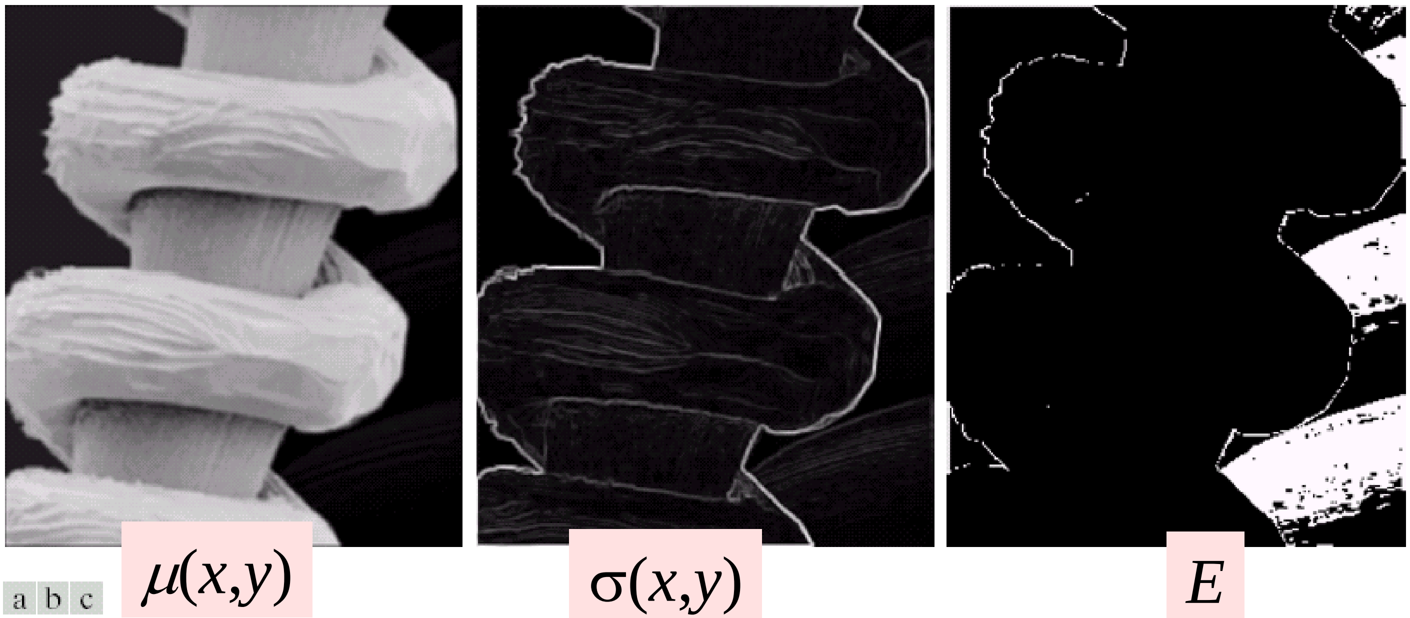


FIGURE 3.25 (a) Image formed from all local means obtained from Fig. 3.24 using Eq. (3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq. (3.3-22). (c) Image formed from all multiplication constants used to produce the enhanced image shown in Fig. 3.26.

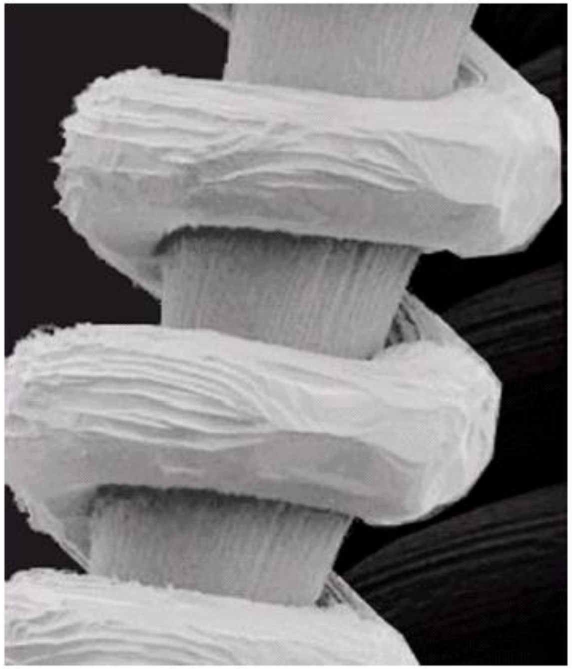
Image

125	135	145	135	..
168	175	158	149	..
210	231	215	129	..
187	192	145	200	..

Mean

174	164
187	

Histogram Statistics for Image Enhanc.



Original Image

Enhanced Image

Enhancement Using Arithmetic/Logic Op.

Arithmetic/logic operations involving images are performed on a pixel-by-pixel basis between two or more images.

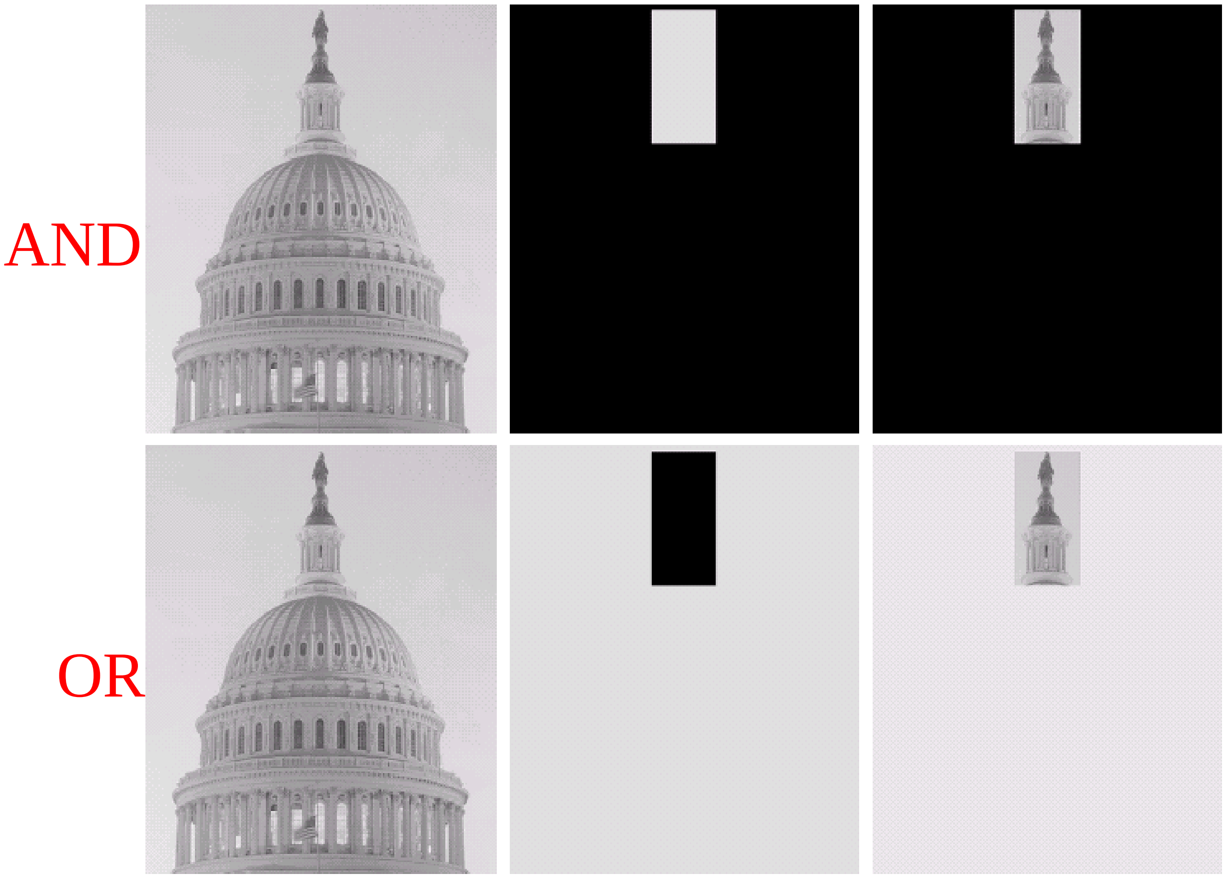
Arithmetic Operations

Addition, Subtraction, Multiplication, and Division

Logic Operations

AND, OR, NOT

Enhancement Using **AND** *and* **OR** *Logic Op.*



a	b	c
d	e	f

FIGURE 3.27 (a) Original image. (b) AND image mask. (c) Result of the AND operation on images (a) and (b). (d) Original image. (e) OR image mask. (f) Result of operation OR on images (d) and (e).

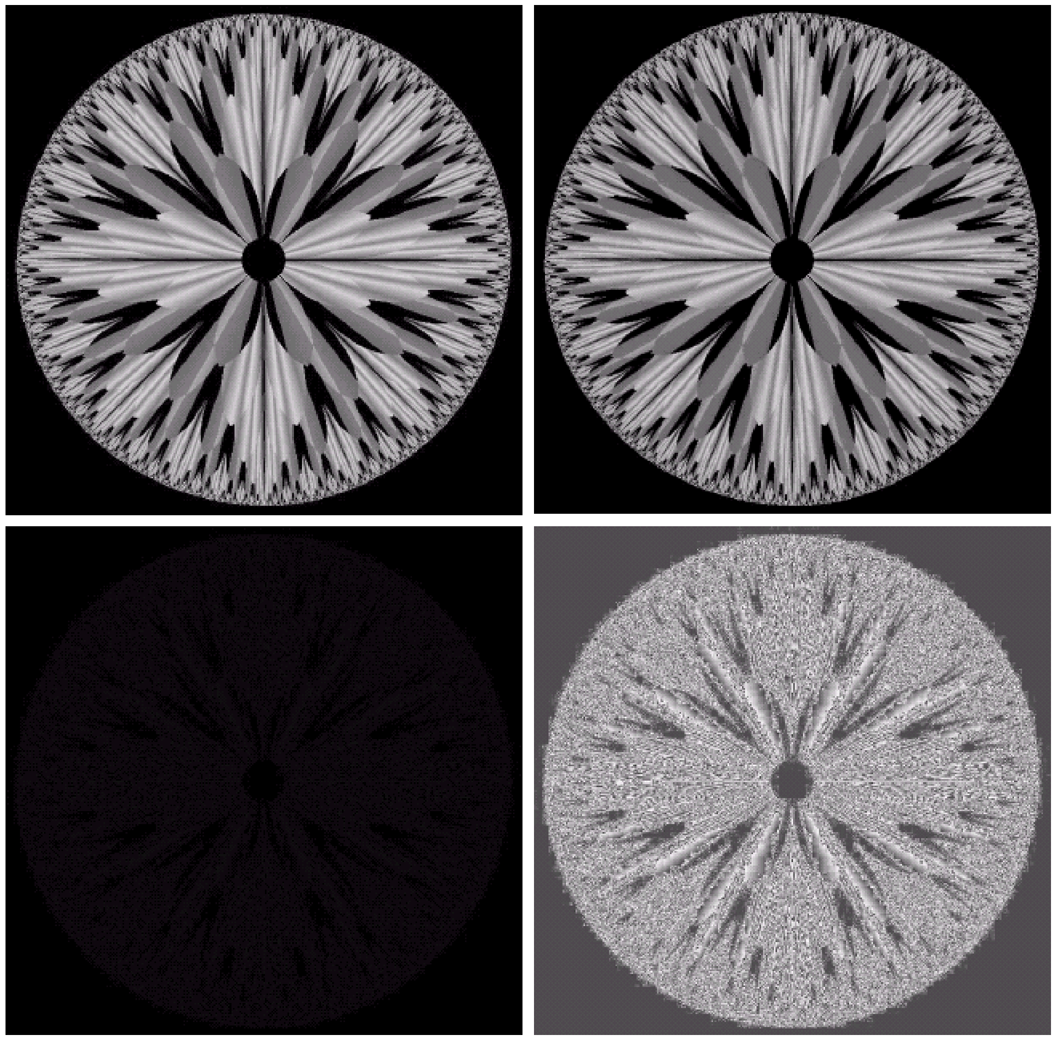
Logic operations are performed on the binary representation of the pixel intensities

Enhancement Using Arithmetic Op. _ **SUB**

a b
c d

FIGURE 3.28
(a) Original fractal image.
(b) Result of setting the four lower-order bit planes to zero.
(c) Difference between (a) and (b).
(d) Histogram-equalized difference image.
(Original image courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA).

Original

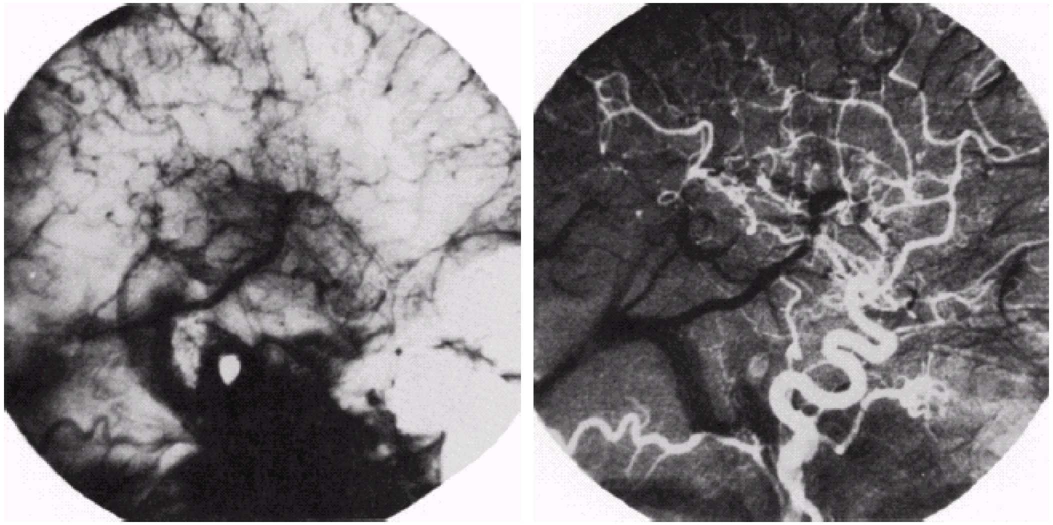


4 Upper-order Bit

Error

HE of Error

Enhancement Using Arithmetic Op. _ **SUB**
Mask Mode Radiography



a b

FIGURE 3.29 Enhancement by image subtraction. (a) Mask image. (b) An image (taken after injection of a contrast medium into the bloodstream) with mask subtracted out.

Problem:

The pixel intensities in the difference image can range from -255 to 255.

Solutions:

1) Add 255 to every pixel and then divide by 2.
2) Add the minimum value of the pixel intensity in the difference image to every pixel and then divide by 255/Max. Max is the maximum pixel value in the modified difference image.

Enhancement Using Arithmetic Op.

Averaging

Original image → $f(x,y)$; Noise with zero mean → $\eta(x,y)$

$$g(x,y) = f(x,y) + \eta(x,y)$$

$$\bar{g}(x,y) = \frac{1}{k}\sum_{i=1}^{k} g_i(x,y)$$

$$E\left[\bar{g}(x,y)\right] = f(x,y)$$

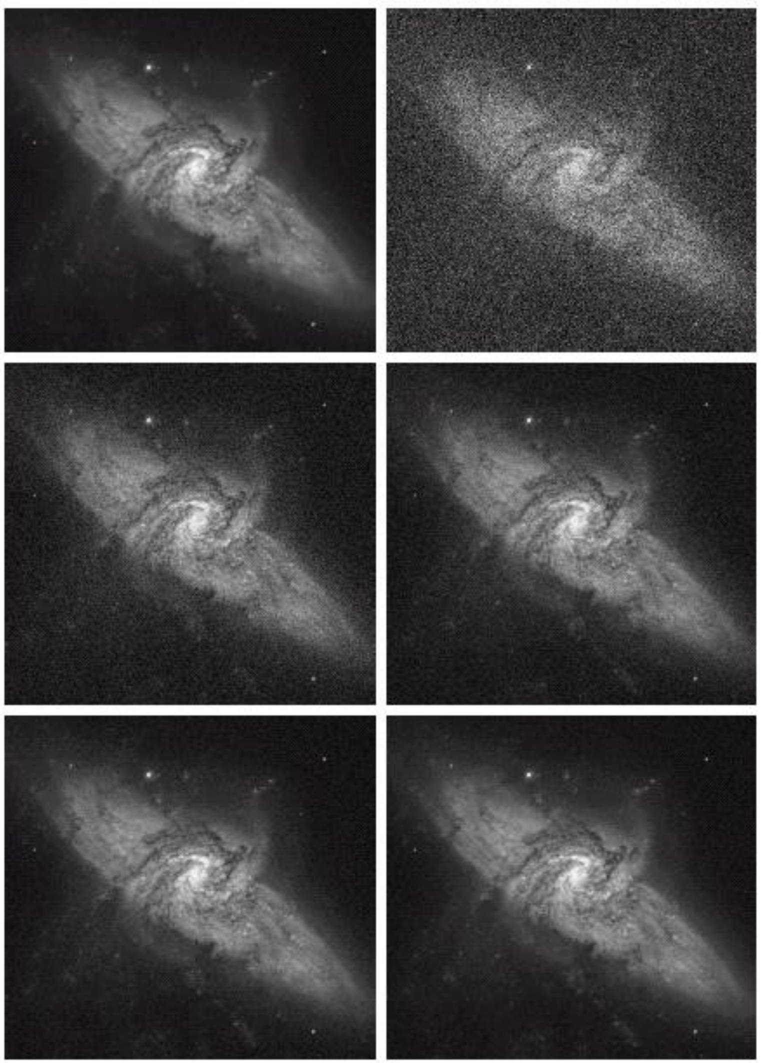
$$\sigma^2_{\bar{g}(x,y)} = \frac{1}{k}\sigma^2_{\eta(x,y)}$$

Enhancement Using Arithmetic Op.
Averaging

$f(x,y)$ $g(x,y)$

$K = 8$ $K = 16$

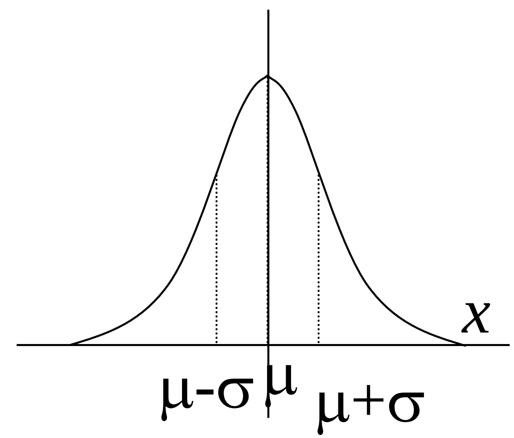
$K = 32$ $K = 128$



a b
c d
e f

FIGURE 3.30 (a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging $K = 8, 16, 64$, and 128 noisy images. (Original image courtesy of NASA.)

Gaussian Noise
mean = 0
variance = 64



Enhancement Using Arithmetic Op.

Averaging

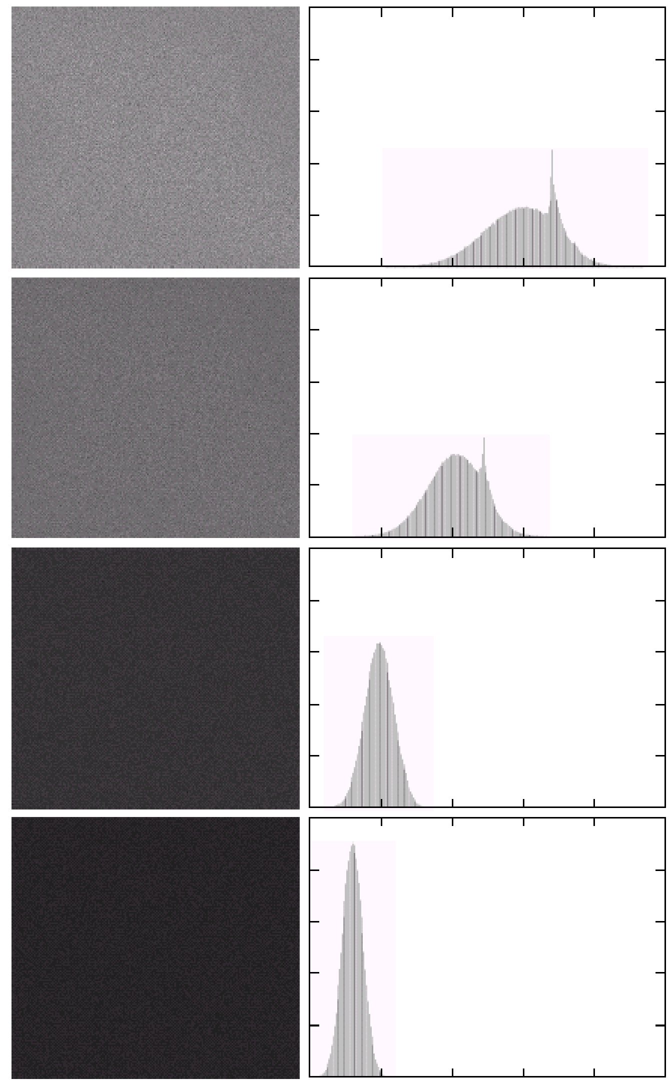
Difference images between original image and images obtained from averaging.

$K = 8$

$K = 16$

$K = 32$

$K = 128$

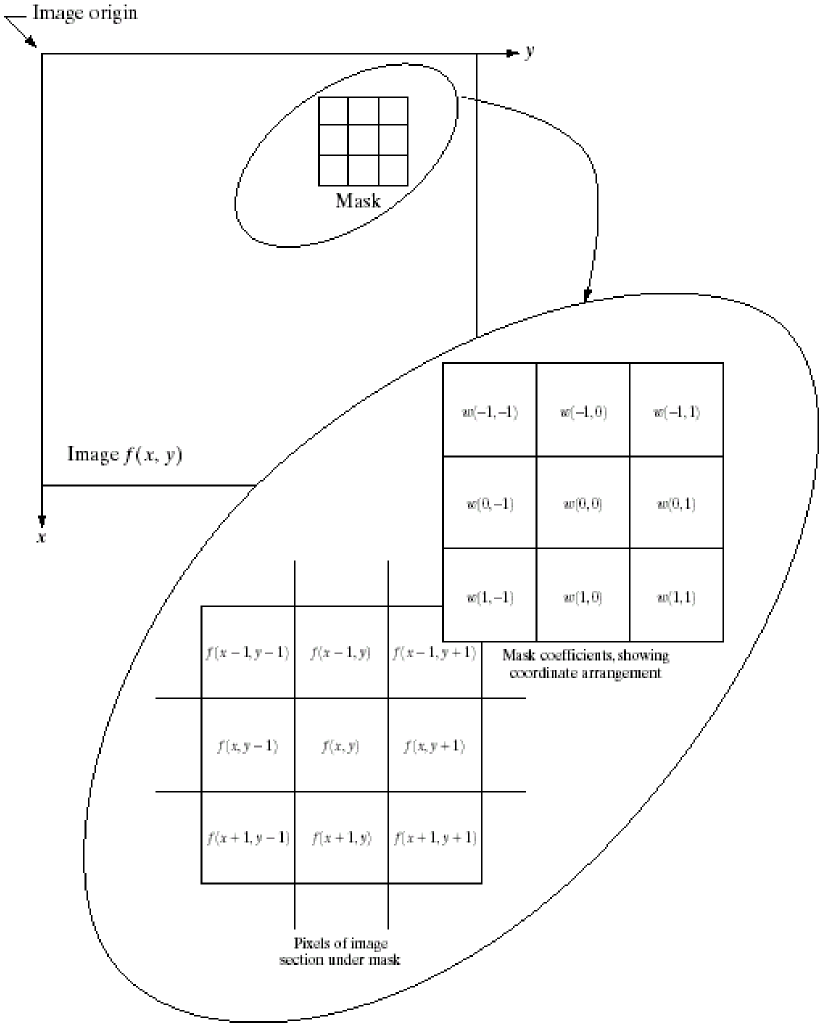


a b

FIGURE 3.31 (a) From top to bottom: Difference images between Fig. 3.30(a) and the four images in Figs. 3.30(c) through (f), respectively. (b) Corresponding histograms.

Notice the mean and variance of the difference images decrease as K increases.

Basics of Spatial Filtering - Linear



Spatial filtering are filtering operations performed on the pixel intensities of an image and not on the frequency components of the image.

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

$$a = (m - 1) / 2 \qquad b = (n - 1) / 2$$

Basics of Spatial Filtering

Response, R, of an $m \times n$ mask at any point (x, y)

$$R = \sum_{i=1}^{mn} w_i z_i$$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Special consideration is given when the center of the filter approach the boarder of the image.

Nonlinear of Spatial Filtering

Nonlinear spatial filters operate on neighborhoods, and the mechanics of sliding a mask past an image are the same as was just outlined. In general however, the filtering operation is based conditionally on the values of the pixel in the neighborhood under consideration, and they do not explicitly use coefficients in the sum-of products manner described previously.

Example

Computation for the median is a nonlinear operation.

Smoothing Spatial Filtering - Linear

Averaging (low-pass) Filters

Smoothing filters are used

- Noise reduction
- Smoothing of false contours
- Reduction of irrelevant detail

Undesirable side effect of smoothing filters

- Blur edges

Weighted average filter reduces blurring in the smoothing process.

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Box filter

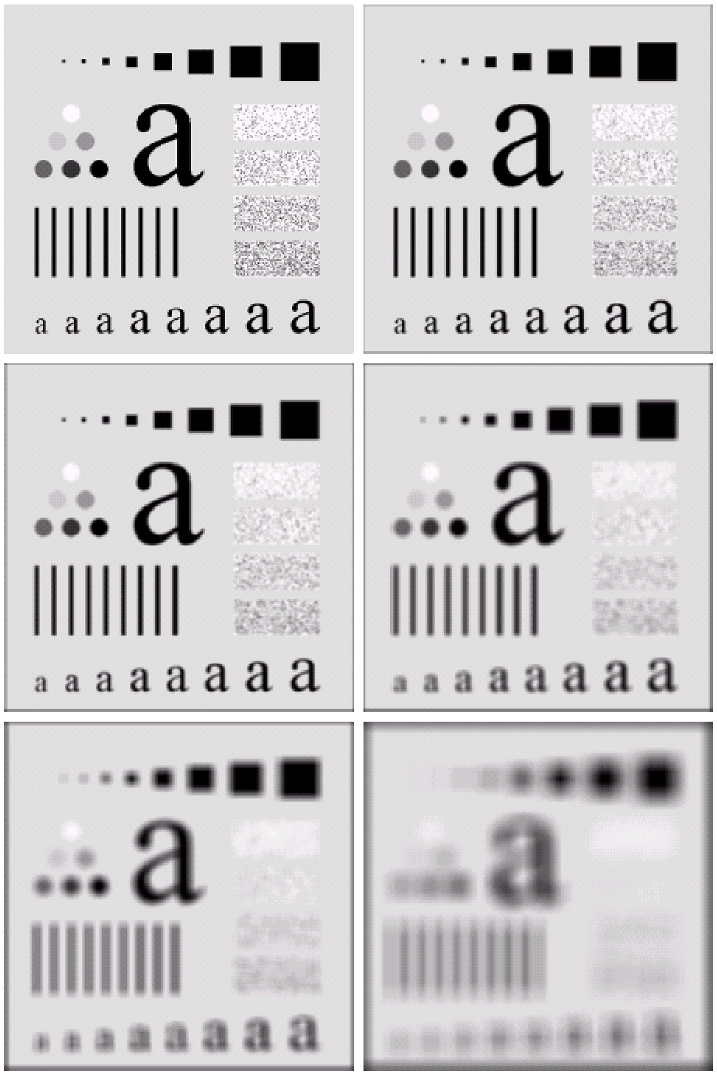
Weighted average

a b

FIGURE 3.34 Two 3 × 3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to the sum of the values of its coefficients, as is required to compute an average.

Smoothing Spatial Filtering _ Linear
Averaging (low-pass) Filters

n = filter size



$n = 3$

$n = 5$

$n = 9$

$n = 15$

$n = 35$

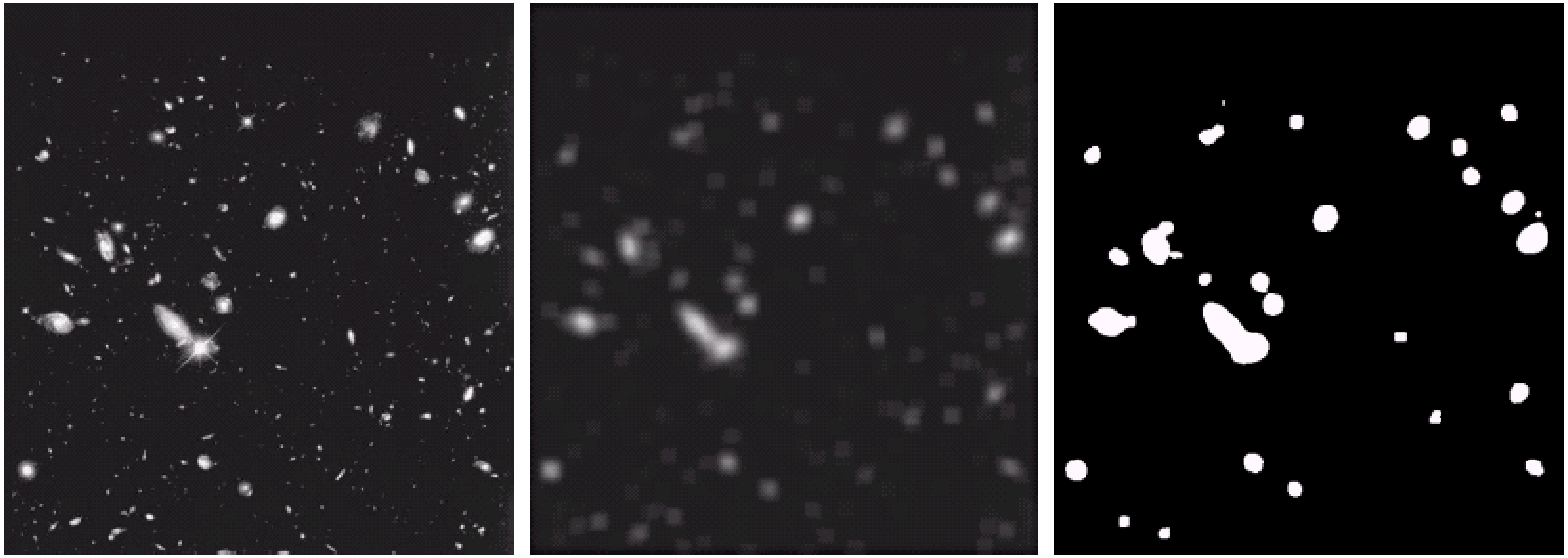
a	b
c	d
e	f

FIGURE 3.35 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

Smoothing Spatial Filtering
Averaging & Threshold

filter size
$n = 15$

Thrsh = 25% of
highest intensity



a b c

FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Smoothing Spatial Filtering
Order Statistic Filters

Order-statistics filters are nonlinear spatial filters whose response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter, and then replacing the value of the center pixel with the value determined by the ranking result.

3×3 Median filter [10 125 125 135 141 141 144 230 240] = 141
3×3 Max filter [10 125 125 135 141 141 144 230 240] = 240
3×3 Min filter [10 125 125 135 141 141 144 230 240] = 10

Median filter eliminates isolated clusters of pixels that are light or dark with respect to their neighbors, and whose area is less than $n^2/2$.

Order Statistic Filters

$n = 3$
Average
filter

$n = 3$
Median
filter

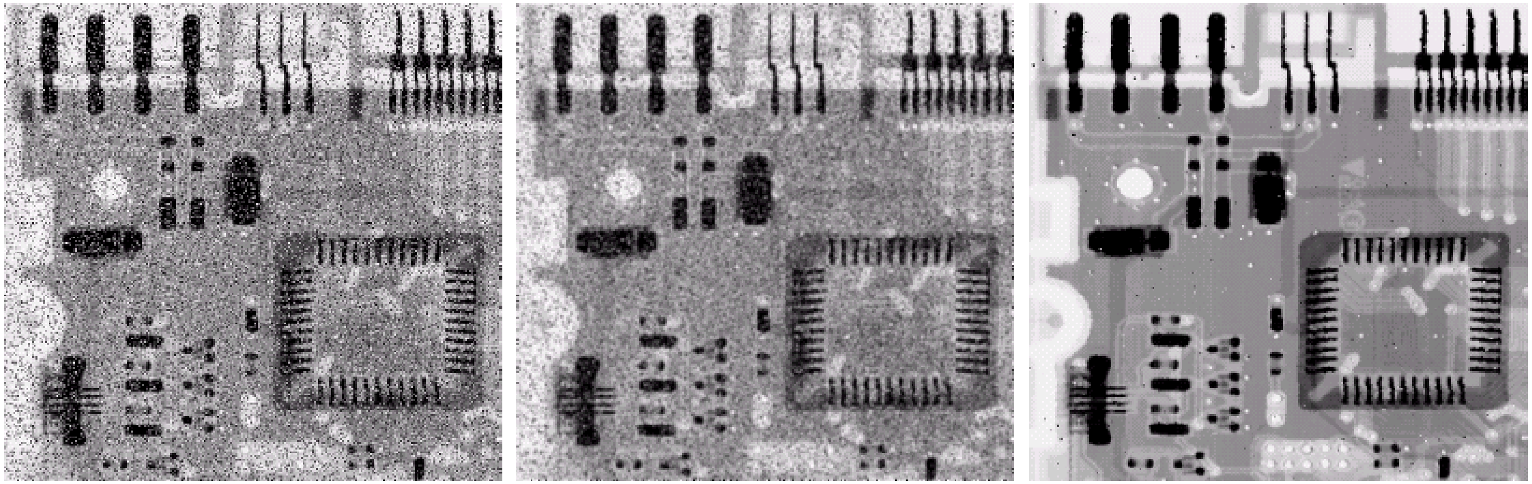


FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Sharpening Spatial Filters

The principal objective of sharpening is to highlight fine detail in an image or to enhance detail that has been blurred.

Image →

$$\frac{1}{9}\sum_{1}^{9} z_i$$

$$\frac{1}{9}\int_{1}^{9} z$$

→ Blurred Image

The derivatives of a digital function are defined in terms of differences.

Sharpening Spatial Filters

Requirements for digital derivative

First derivative

1) Must be zero in flat segment
2) Must be nonzero along ramps.
3) Must be nonzero at the onset of a gray-level step or ramp

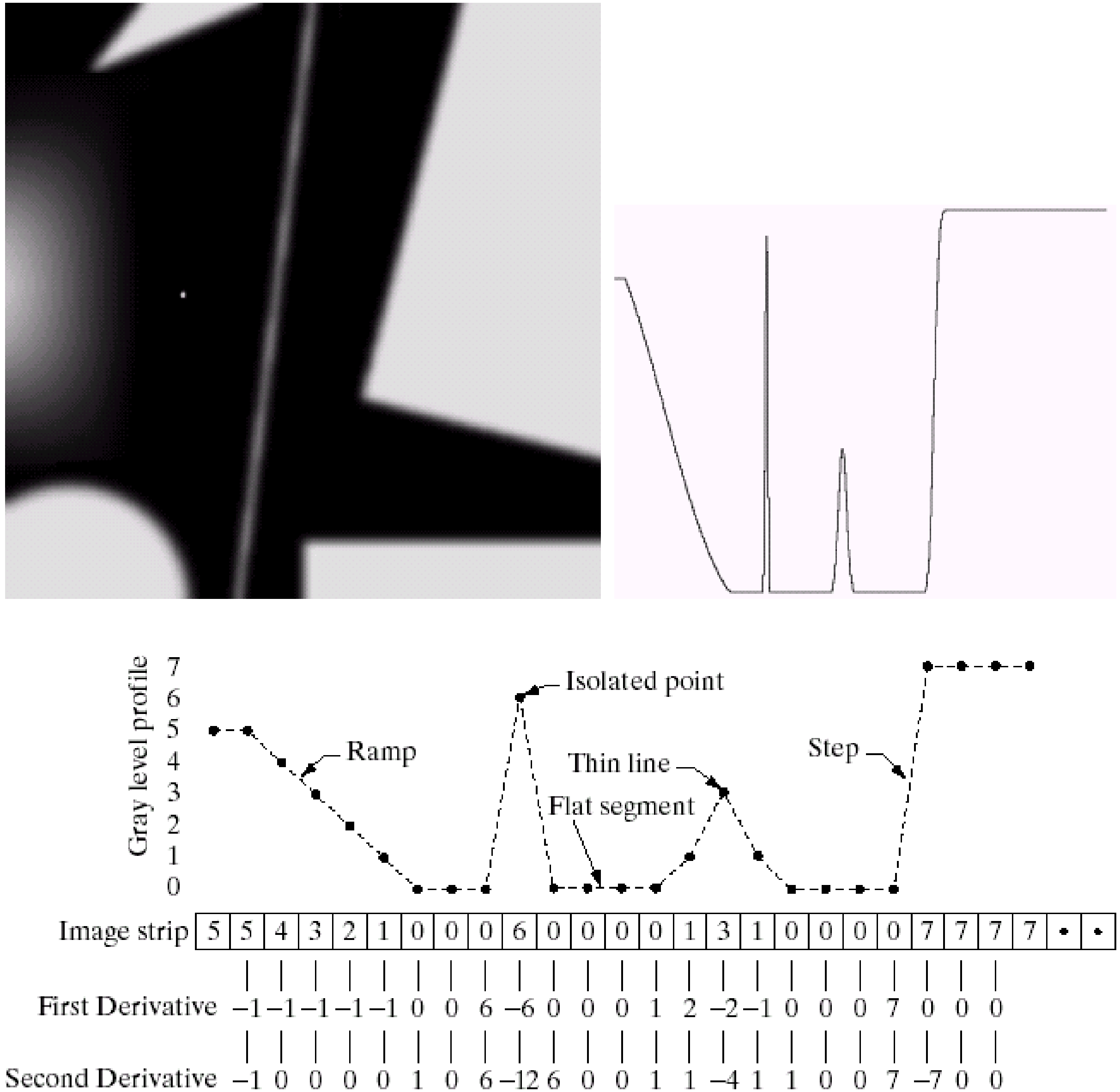
Second derivative

1) Must be zero in flat segment
2) Must be zero along ramps.
3) Must be nonzero at the onset and end of a gray-level step or ramp

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Sharpening Spatial Filters



a b
c

FIGURE 3.38 (a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point. (c) Simplified profile (the points are joined by dashed lines to simplify interpretation).

Sharpening Spatial Filters

Comparing the response between first- and second-ordered derivatives:
1) First-order derivative produce thicker edge
2) Second-order derivative have a stronger response to fine detail, such as thin lines and isolated points.
3) First-order derivatives generally have a stronger response to a gray-level step {2 4 15}
4) Second-order derivatives produce a double response at step changes in gray level.

In general the second derivative is better than the first derivative for image enhancement. The principle use of first derivative is for edge extraction.

Use of First Derivative for Edge Extraction

Gradient

First derivatives in image processing are implemented using the magnitude of the gradient.

$$\nabla \mathbf{f} = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix}^t$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$$\nabla f = mag(\nabla \mathbf{f}) = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{0.5} \approx |G_x| + |G_y|$$

Roberts operator

$G_x = (z_9 - z_5)$ and $G_y = (z_8 - z_6)$

Sobel operator

$G_x = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$ and

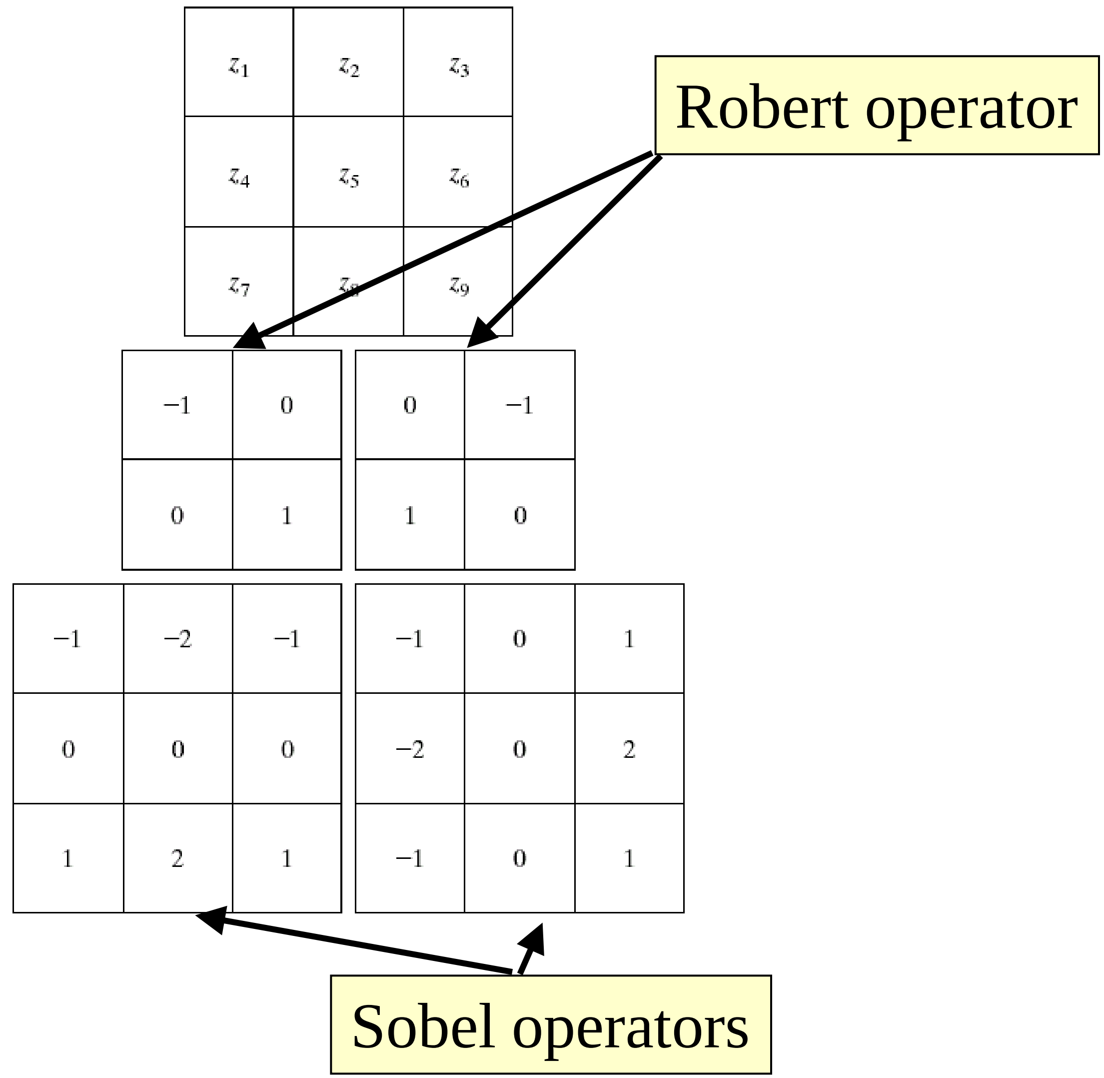
$G_y = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$

Use of First Derivative for Edge Extraction

Gradient

a
b c
d e

FIGURE 3.44
A 3 × 3 region of an image (the z's are gray-level values) and masks used to compute the gradient at point labeled z_5. All masks coefficients sum to zero, as expected of a derivative operator.



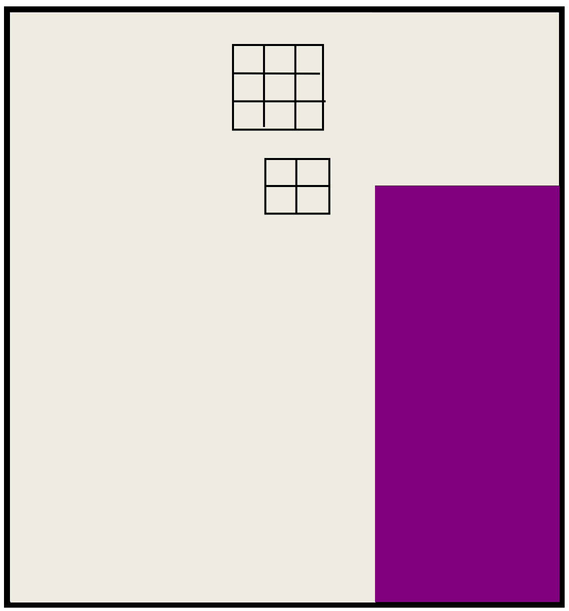
Use of First Derivative for Edge Extraction

Gradient

-1	0	1
-2	0	2
-1	0	1

-1	-2	-1
0	0	0
1	2	1

$f(x,y) = [40, 140]$



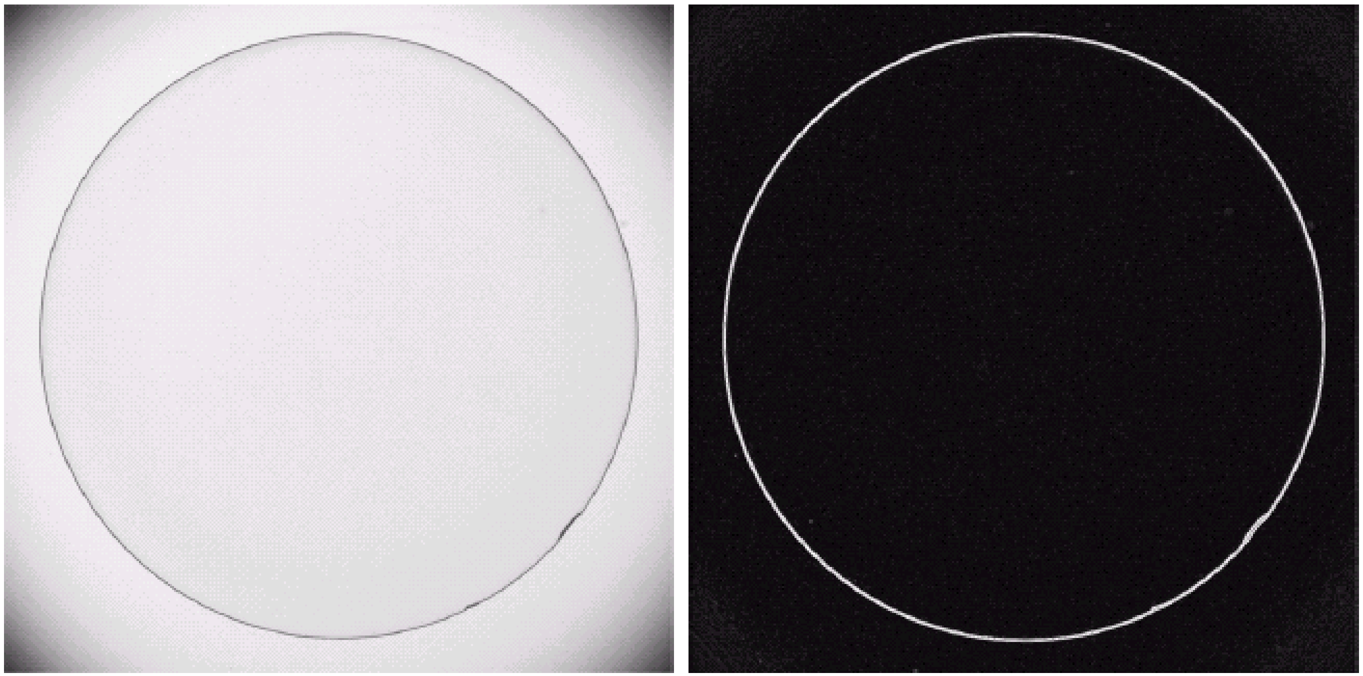
…. 0	0	0	0	0	0		
…. 0	200	400	400	400	400	……	
…. 0	400	600	400	400	400	…...	
…. 0	400	400	0	0	0		
…. 0	400	400	0	0	0		
…. 0	400	400	0	0	0		

… 0	0	0	0	0	
… 0	100	200	200	200	
… 0	200	0	0	0	
… 0	200	0	0	0	

	-1
1	

-1	
	1

Use of First Derivative for Edge Extraction
Gradient

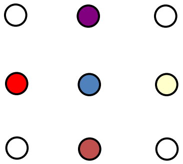


a b

FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

2nd Derivative _ Laplacian

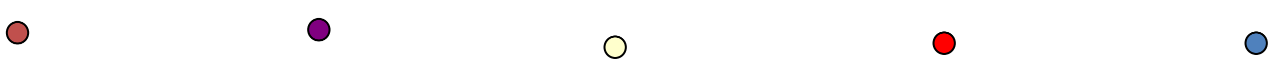
$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$



$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(y, y)]$$



Use of 2nd Derivative for Enhancement
Laplacian

Isotropic filter response is independent of the direction of the discontinuities in the image to which the filter is applied.

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

(a)

0	1	0
1	−4	1
0	1	0

(b)

1	1	1
1	−8	1
1	1	1

(c)

0	−1	0
−1	4	−1
0	−1	0

(d)

−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

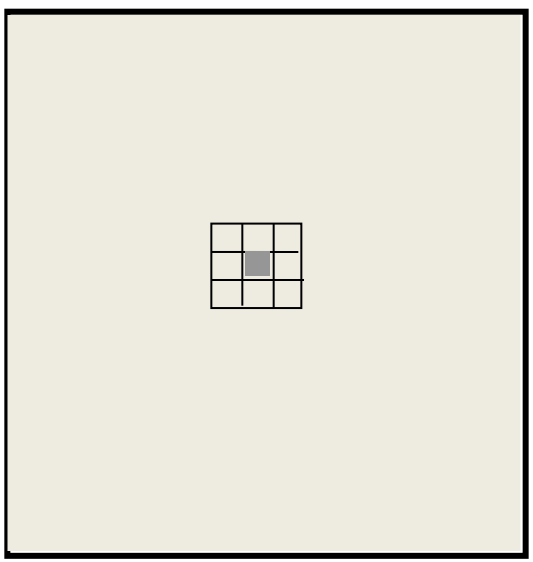
FIGURE 3.39 (a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

Use of 2nd Derivative for Enhancement

Laplacian

0	1	0
1	-4	1
0	1	0

$f(x,y) = [90, 100]$



…	0	0	0	0	…
…	0	10	0	0	…
…	10	-40	10	0	…
…	0	10	0	0	…
…	0	0	0	0	…

Use of 2nd Derivative for Enhancement

Laplacian

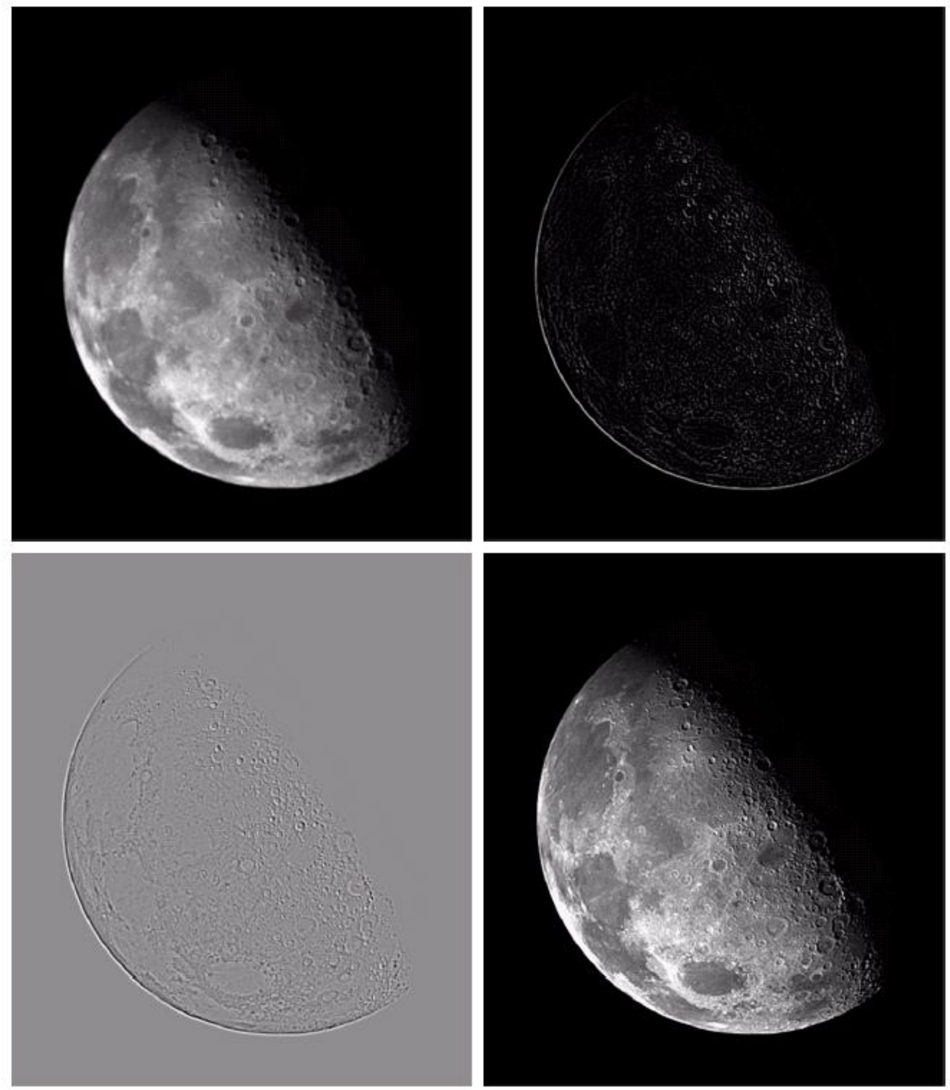
1	1	1
1	-8	1
1	1	1

If the center coefficient of the laplacian mask is negative

$$g(x,y)=\begin{cases} f(x,y)-\nabla^2 f(x,y) \\ f(x,y)+\nabla^2 f(x,y) \end{cases}$$

a b
c d

FIGURE 3.40 (a) Image of the North Pole of the moon. (b) Laplacian-filtered image. (c) Laplacian image scaled for display purposes. (d) Image enhanced by using Eq. (3.7-5). (Original image courtesy of NASA.)

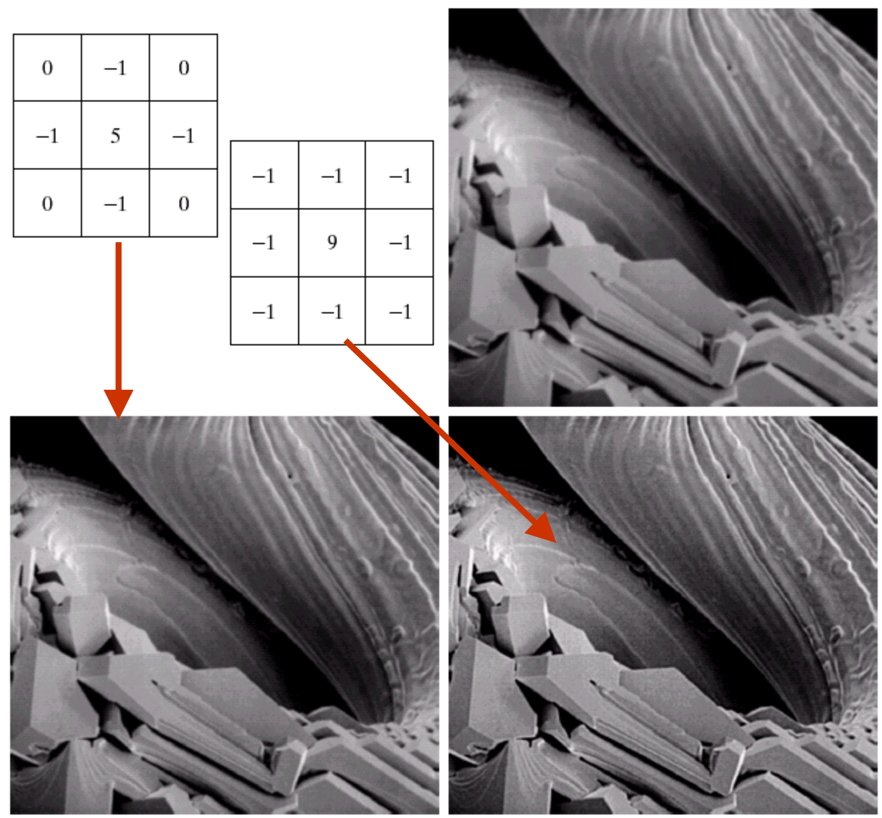


Use of 2nd Derivative for Enhancement
Laplacian

$$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)]$$

$$g(x,y) = f(x,y) - \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} =$$

$$\nabla^2 f(x,y)$$



0	–1	0
–1	5	–1
0	–1	0

–1	–1	–1
–1	9	–1
–1	–1	–1

a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Un-sharp Masking and High-boost Filtering

High-boost filtering is used when the original image is blurred and dark.

$$f_{hb} = Af(x,y) - \nabla^2 f(x,y) \qquad A > 1$$

0	−1	0
−1	$A + 4$	−1
0	−1	0

−1	−1	−1
−1	$A + 8$	−1
−1	−1	−1

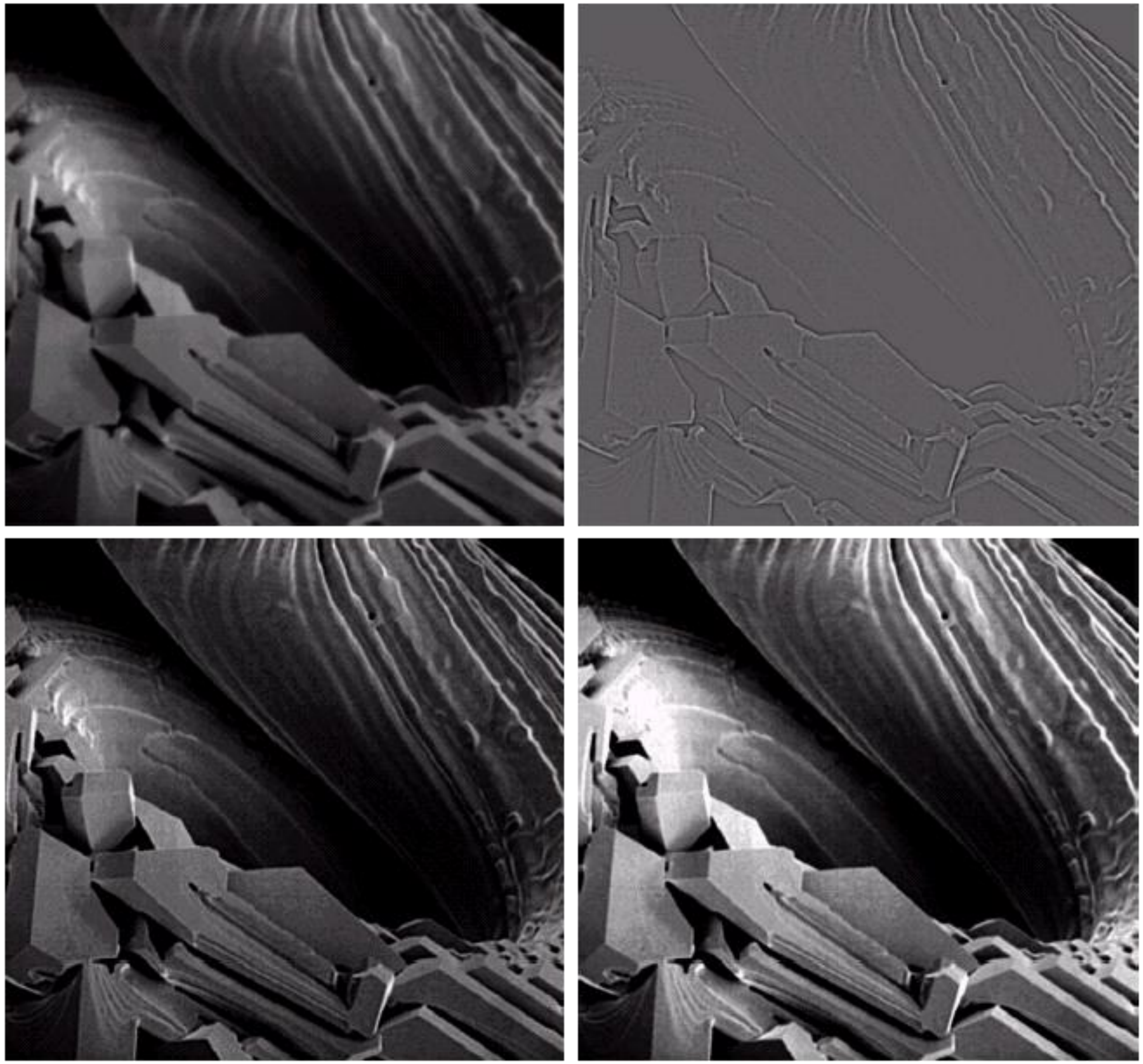
a b

FIGURE 3.42 The high-boost filtering technique can be implemented with either one of these masks, with $A \geq 1$.

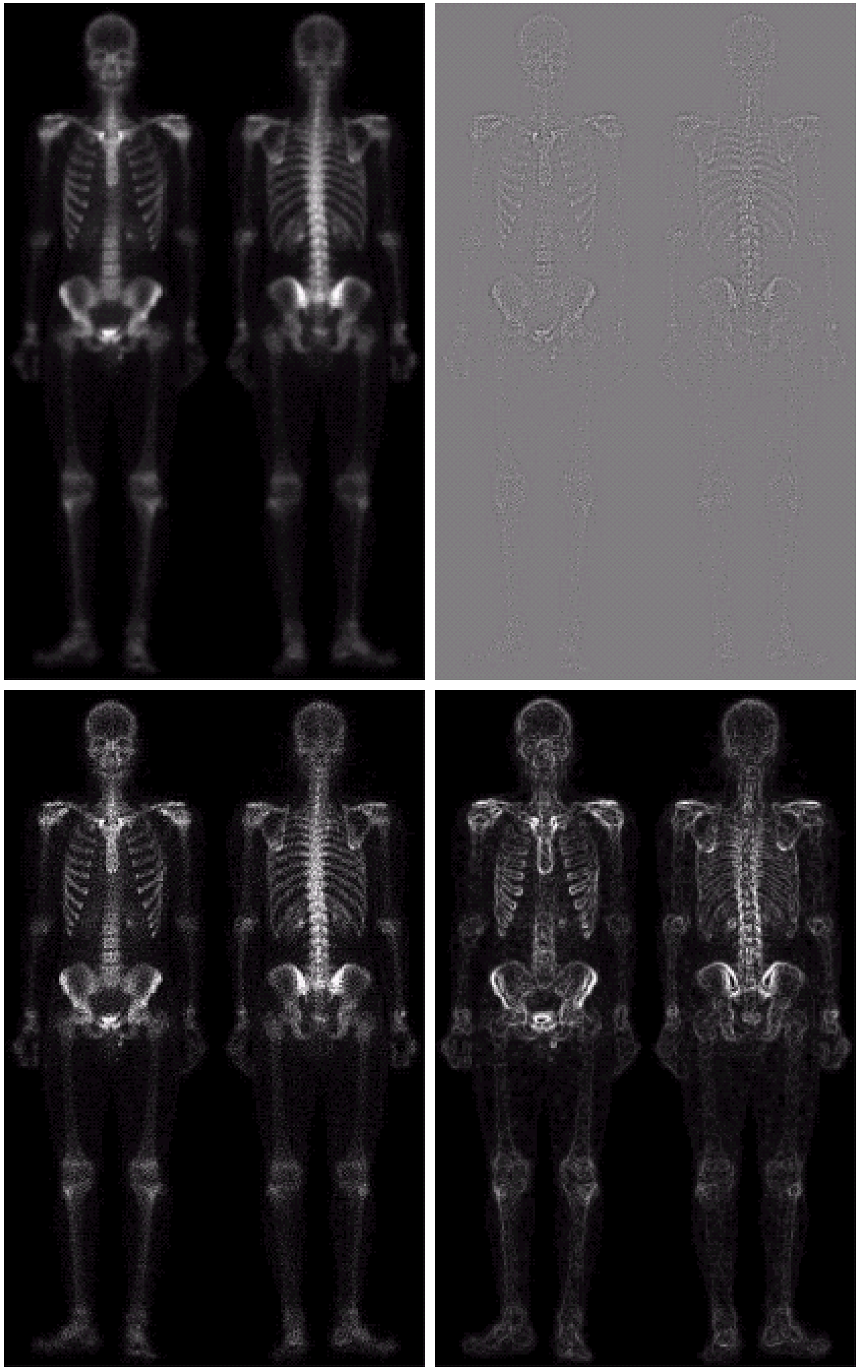
Un-sharp Masking and High-boost Filtering

a	b
c	d

FIGURE 3.43 (a) Same as Fig. 3.41(c), but darker. (b) ← (a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$. (c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.



Combining Spatial Enhancement Methods



a	b
c	d

FIGURE 3.46
(a) Image of whole body bone scan.
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel of (a).

Combining Spatial Enhancement Methods

e	f
g	h

FIGURE 3.46
(Continued)
(e) Sobel image smoothed with a 5 × 5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)